Working Paper # 23
Revision of MS & PhD Mathematics Programs
Sponsored by SNS

MS Mathematics
Program Description

1. The MS degree in Mathematics focuses on strengthening the ability of a student in mathematical reasoning and logical thinking. Students in this program prepare themselves either for their further development in the field of Mathematics or for jobs in academic, industrial, business and government organizations. The school offers a wide range of courses from its major thrust areas, which include Analysis, Algebra, Topology, Differential Equations, Mathematical Physics, Discrete Mathematics, Fluid Dynamics, and Computational Mathematics.

The MS Mathematics students are required to complete 24 credit hours of coursework including 12 credit hours of core courses. The existing set of core courses includes MATH-801, MATH-802, MATH-803 and MATH-804. The revised set of core courses comprises MATH-801, MATH- 803, MATH-XXX (Computational Mathematics) and MATH- 817. MATH-802 and MATH-804 from the existing program are replaced with MATH-XXX (Computational Mathematics) and MATH-817. The remaining 12 credit hours are required to be completed from elective courses. The MS Mathematics program also includes 6 credits hours for the thesis.

Rationale for Revision

2. MPhil leading to PhD program was started in 2004. In 2012, the structure of the program was revised and separate programs at MS and PhD levels were introduced.

As per NUST Policy, all programs are required to be revised after the completion of every 4 years. The existing MS curriculum was revised in 2019. Therefore, the current revision is initiated keeping in view the national and international practices. In this revision, the core of MS program and the contents of some courses are revised. Also, some new courses are included. Scheme of Studies for MS Mathematics is presented as follows:

3. **Eligibility Criteria**: In addition to NUST laid down criteria:

MSc or BS (16 years of education) in Mathematics or equivalent.

Semester-wise Breakdown

First Year					
Semester-I			Semester-II		
Course Code	**Course Title**	**Credits**	**Course Code**	**Course Title**	**Credits**
MATH 801	Algebra	3-0		Elective-I	3-0
MATH 803	Geometry	3-0		Elective-II	3-0
MATH-XXX	Computational Mathematics	3-0		Elective-III	3-0
MATH-817	Advanced Functional Analysis	3-0		Elective-IV	3-0
Total Credits		**12**	**Total Credits**		**12**

Second Year		
Course Code	**Course Title**	**Credits**
RM-898	Research Methodology*	2-0
SEM/WKSP-897	Seminar/Workshop*	1-0
MATH-899	MS Thesis	6-0

*Additional Course

Timeframe of commencement

3. The revised MS Mathematics program will be implemented for Fall 2023 and onward batches.

Input from industry and Academia

4. Input on the revised curriculum has been sought from the following academia and industry representatives in an advisory board meeting held on October 24, 2022.

S.No	Name	Designation/ Institution
1	Prof. Dr. Muhammad Sajid	Professor of Department of Mathematics IIU Islamabad
2	Prof. Dr. Shahid Hamid	Professor/ Dean of Natural Sciences, QAU Islamabad.
3	Mr. Tariq Mehmood Khan	CEO Redox (SMC PVT) LTD Islamabad

5. Minutes of the advisory board meeting are attached. Suggestions/inputs from the following alumnae have been incorporated in the working paper.

a. Ghafirlia Istafa
b. Zain ul Abdeen
c. Hafiz Muhammad Fahad

Approved by DBS/FBS

6. Approved by FBS held on November 4th, 2022.

7. **Changes in MS Mathematics Courses**

Summary of change

S. No	Type of Change	No. of courses
1.	Courses revised	02
2.	Courses with no change	36
3.	New courses included	03
4.	Courses discarded (replaced with new courses)	03

Details of Changes

S. No	Code	Subject	CHs	Details of Changes			
				Code change	Title change	Contents revised	Remarks
Revised courses							
1	MATH-801	Algebra	3-0	No	No	Yes	Contents revised
2	MATH-807	Commutative Algebra	3-0	No	No	Yes	Contents revised
Courses with no change							
1.	MATH-803	Geometry	3-0	-	-	-	
2.	MATH-817	Advanced Functional Analysis	3-0	-	-	-	
3.	MATH-818	Theory of Ordinary Differential Equations	3-0	-	-	-	
4.	MATH-819	Analysis of Fractional Differential Equations	3-0	-	-	-	
5.	MATH-820	Calculus of Variations and Optimal Control	3-0	-	-	-	
6.	MATH-821	Analytical Approximate Solutions of ODEs	3-0	-	-	-	
7.	MATH-822	Mathematical Modelling-I	3-0	-	-	-	
8.	MATH-903	Partial Differential Equation-I	3-0	-	-	-	
9.	MATH-905	Symmetry Methods for Differential Equation-I	3-0	-	-	-	
10.	MATH-908	Fixed Point Theory	3-0	-	-	-	
11.	MATH-909	Continuum Mechanics-I	3-0	-	-	-	
12.	MATH-911	Special Function	3-0	-	-	-	
13.	MATH-941	Graph Theory	3-0	-	-	-	
14.	MATH-943	Convex Analysis	3-0	-	-	-	
15.	MATH-944	Semigroup Theory of Operators	3-0	-	-	-	
16.	MATH-945	Lie Group Representations	3-0	-	-	-	
17.	MATH-946	Category Theory	3-0	-	-	-	
18.	MATH-949	Combinatorics	3-0	-	-	-	
19.	MATH-955	General Relativity and Cosmology	3-0	-	-	-	
20.	MATH-956	Finite Volume Method	3-0	-	-	-	
21.	MATH-957	Algebraic Topology	3-0	-	-	-	
22.	ME-881	Advanced Fluid Mechanics	3-0	-	-	-	
23.	PHY-801	Classical Mechanics	3-0				
24.	PHY-803	Quantum Mechanics	3-0				
25.	PHY-805	Electromagnetism	3-0				
26.	PHY-806	Thermal Physics	3-0				
27.	PHY-902	Quantum Field Theory-I	3-0				
28.	PHY-907	General Relativity	3-0				
29.	PHY-908	Cosmology-I	3-0				
30.	PHY-912	Relativistic Astrophysics	3-0				
31.	PHY-914	Particle Physics-I	3-0				

32.	PHY-920	Classical Field Theory	3-0				
33.	STAT-806	Statistical Learning	3-0	-	-	-	
34.	RM-898	Research Methodology	2-0	-	-	-	
35.	SEM/WKS P-897	Seminar/Workshop	1-0	-	-	-	
36.	MATH-899	MS Thesis	6-0	-	-	-	
New courses included							
1	MATH-XXX	Computational Mathematics	3-0	-	-	-	Replacement of MATH-804 Differential Equations
2	MATH-XXX	Advanced Topology	3-0	-	-	-	-
3	MATH-XXX	Finite Difference Methods for Differential Equations	3-0	-	-	-	
Courses discarded							
Core Courses							
1	MATH-804	Differential Equations	3-0	-	-	-	Replaced with MATH-XXX Computational Mathematics
2	MATH-802	Analysis	3-0	-	-	-	Replaced with already approved elective course MATH-817 Advanced Functional Analysis
Elective courses							
1	MATH-XXX	Numerical Methods for Differential Equations	3-0	-	-	-	Replaced with MATH-XXX Finite Difference Method for Differential Equations

8. Detailed Course Contents -MS Mathematics attached at **Annex A** and detail & course content - PhD. Mathematics Program at **Annex B**.

9. Revised List of MS Mathematics Courses with Prerequisites attached at **Annex C** and Revised List of Ph.D. Mathematics Courses with Prerequisites attached at **Annex D**.

Comments of Academics Directorate

10. No additional requirement of faculty, classrooms & labs. The proposal was deliberated and endorsed by UCRC held on 15 Dec 2022.

Recommendation of Academics Directorate

11. Revision of MS & PhD Mathematics at SNS are recommended for approval w.e.f Fall 2023.

12. Academic council is requested for the decision.

Annex A

Detailed Course Contents -MS Mathematics

MATH-801 Algebra

Credit Hours: 3-0
Prerequisite: None

Objectives and Goals: This course will provide a base for many subjects in modern Algebra such as commutative algebra, algebraic geometry, algebraic number theory, computational commutative algebra, multiplicative ideal theory, homological algebra and combinatorial commutative algebra and representation theory.

Core Contents: Groups, group actions and permutation representations, class equation of group, Sylow's theorems, simple groups, direct product and finitely generated abelian groups, rings, ideals, Euclidean domains, principal ideal domains, unique factorization domains.

Detailed Course Contents: Groups, dihedral groups, symmetric groups, matrix groups, the Quaternion group, homomorphism and isomorphism, subgroups generated by a subset of a group, the lattice of subgroups of a group, Fibers of a group homomorphism, quotient groups, group actions, group actions and permutation representations, group acting on themselves by left multiplication, group acting on themselves by conjugation, the class equation, the Sylow's theorems, simple groups, conjugacy in symmetric groups, the fundamental theorem of finitely generated abelian groups, rings, subrings, ideals, polynomial rings, quotient rings, ring homomorphism, properties of ideals, integral domains, prime and irreducible elements, Euclidean domains, principal ideal domains, unique factorization domains, polynomial rings over fields, polynomial rings that are unique factorizations

Course Outcomes: On successful completion of this course, students will know groups, subgroups, group action, factor groups, number of all possible abelian groups up to isomorphism for a given order, Sylow theorems, application to simplicity of groups, rings, subrings, ideals, polynomial rings, reducible and irreducible polynomials over certain rings, integral domains, Euclidean domains, principal ideal domains, unique factorization domains

Textbook: David S. Dummit, Richard M. Foote, Abstract Algebra, 3rd Ed., John Wiley &Sons.

Reference books

1. N. Herstein, Topics in Algebra, John Wiley and Sons.
2. W. Keith Nicholson, Introduction to Abstract Algebra, (3rd edition), 2007, John Wiley & son

Weekly Breakdown		
Week	*Section*	*Topics*
1	Sec. 1.2-1.6	Group of symmetries of a geometric object, examples, presentation of a group, matrix groups, the quaternion group, group homomorphisms
2	Sec. 1.7	Group actions, examples of group actions, permutation representation associated to the given action. faithful and transitive actions.
3	Sec. 2.1, 2.2, 2.3	Subgroups, centralizers, and normalizers. Stabilizers and kernels of the group actions. Cyclic groups and cyclic subgroups.
4	Sec. 2.4, 2.5	Subgroups generated by a subgroup of a group, the lattice of the subgroups of a group.
5	Sec. 3.1	Fibers of a group homomorphism and related theorems, quotient group using fibers of a group homomorphism, quotient group by a normal subgroup.
6	Sec. 3.2	Lagrange theorem and its converse, Cauchy's theorem, composition of two subgroups and related results.
7	Sec. 3.3, 4.1	Isomorphism theorems, the correspondence theorem and its applications to factor group. Group action and permutation representations.
8	Sec. 4.2	Orbit stabilizer theorem, group acting on themselves by left multiplication, Smallest prime index theorem.
9	**Mid Semester Exam**	
10	Sec. 4.3	Group acting on themselves by conjugation, the class equation of a group and applications, conjugacy in S_n.
11	Sec. 4.5	Proofs of Sylow's theorems using group action, applications of Sylow's theorems to simple groups.
12	Sec. 5.1, 5.2	Direct products, the fundamental theorem for finitely generated abelian groups.
13	Sec. 7.1, 7.2	Rings, matrix ring, group ring, the ring of residue classes modulo n, polynomial ring in several variables, integral domains, fields.
14	Sec. 7.3	Ideals, quotient rings, ring homomorphism, isomorphism theorems for rings, the correspondence theorem for rings and applications to quotient rings.
15	Sec. 7.4, 8.1, 8.2	Properties of ideals, characterization of prime and maximal ideals. Norms on integral domains, division algorithms for integral domains, examples, principal ideal domains, examples.
16	Sec. 8.3	Prime and irreducible elements, examples, unique factorization domains, examples.
17		Review
18	**End Semester Exam**	

MATH-803 Geometry

Credit Hours: 3-0
Prerequisite: None

Objectives and Goals: After having completed this course, the students would be expected to understand classical concepts in the local theory of curves, surfaces and manifolds. Also the students will be familiar with the geometrical interpretation of the terminology used in the course.

Detailed Course Contents: Curves, Surfaces -Topological Invariants, Geometry on a Surface or Riemannian Geometry, Geodesics, Generalization of the Concept of Tangent and of Tangent Plane, to a Surface Manifolds -Tensor Fields - Covariant Differentiation, Tangent Vectors and Mappings, Tangent or Contravariant" Vectors, Vectors as Differential Operators, The Tangent Space to Mn at a Point, Change of Coordinates, Vector Fields and Flows on Rn, Vector Fields on Manifolds, Functionals and the Dual Space, The Differential of a Function, Scalar Products in Linear Algebra, Riemannian Manifolds and the Gradient Vector, The Tangent Bundle, The Cotangent Bundle and Phase Space, Covariant Tensors, Contravariant Tensors, Mixed Tensor, Properties, The Tensor Product of Covariant Tensors, Wedge Product, The Geometric Meaning, Special Cases, Computations and Vector Analysis, The Exterior Differential, A Coordinate Expression for d, The Pull-Back of a Covariant Tensor,. Integration of a p-Form in Rp, Integration with boundaries, Stokes' theorem, The Lie Bracket,The Lie Derivatives of Forms, Covariant Derivative, Curvature of an Affine Connection, Geodesics.

Course Outcomes: Students are expected to understand classical concepts in the local theory of curves, surfaces and manifolds. Also the students will be familiar with the geometrical interpretation of the terminology used in the course. Students will be able to apply learned concepts in other related fields.

Text Books:
T. Frankel, The Geometry of Physics, Cambridge University Press, 2012 (TB2).
A. Visconti, Introductory Differential Geometry for Physicists, World Scientific, 1992 (TB1).

Reference Books:
1. Bernard F. Schutz , Geometrical Methods of Mathematical Physics, Cambridge University Press, 1980.
2. Serge Lang, Fundamentals of Differential Geometry, Springer, 1999.

Weekly Breakdown		
Week	***Section***	***Topics***
1	1(TB2)	Curves, handouts
2	2(TB2)	Surfaces -Topological Invariants,
3	3(TB2)	Geometry on a Surface or Riemannian Geometry
4	4(TB2)	Geodesics
5	5(TB2)	Generalization of the Concept of Tangent and of Tangent Plane to a Surface
6	(TB1,TB2) 1.1a,1.2(a-c) 1.3(a-c)	Manifolds -Tensor Fields - Covariant Differentiation Tangent Vectors and Mappings, Tangent or "Contravariant" Vectors, Vectors as Differential Operators, The Tangent Space to Mn at a Point
7	(TB1)1.4(a-b)	Change of Coordinates, Vector Fields and Flows on Rn, Vector Fields on Manifolds
8	(TB1)2.1(a-d)	Functionals and the Dual Space, The Differential of a Function, Scalar Products in Linear Algebra, Riemannian Manifolds and the Gradient Vector
9	**Mid Semester Exam**	
10	(TB1)2.2a, 2.3(a-b)	The Tangent Bundle, The Cotangent Bundle and Phase Space
11	(TB1)2.4(a-e).	Covariant Tensors, Contravariant Tensors, Mixed Tensor, Properties
12	(TB1)2.5(a-e) 2.6(a-c)	The Exterior Differential, A Coordinate Expression for d,
13	(TB1)2.7a,	The Pull-Back of a Covariant Tensor,
14	3.1, 3.2, 3.3	Integration of a p-Form in Rp, Integration with boundaries, Stokes' theorem
15	(TB1)4.1, 4.2a.	The Lie Bracket, The Lie Derivatives of Forms
16	(TB1)9.1(a-c)	Covariant Derivative, Curvature of an Affine Connection Godesics
17		Review
18	**End Semester Exam**	

MATH-XXX Computational Mathematics

Credit Hours: 3-0
Prerequisite: None
Course Objectives: The main objective of this course is to train students to acquaint with the processes involved in numerical technique. The rigorous analysis of the numerical techniques to solve different problems pertaining to physical processes will be presented. Moreover, the students will get to know the programming sense of numerical procedures.
Core contents: Root finding techniques, Interpolation, Numerical differentiation, Runge-Kutta methods, Higher order method, Boundary value problem, Introduction to finite difference method for ODEs and PDEs
Course Contents: Newton's method for algebraic equations, interpolation and Lagrange polynomial, numerical differentiation, higher order Taylor methods, Runge-Kutta methods, higher-order differential equations and system of equations, the linear and nonlinear shooting methods, introduction to finite difference method for ODEs and PDEs
Course Outcomes: After reading this course one will be able to:

- Understand basics and advanced techniques in numerical methods
- Find solutions of system of nonlinear equations
- Solving IVP and BVP numerically
- Apply finite difference method to partial differential equations

Overview basics of numerical method algorithm and its implementation in software (MATLAB)

Textbook:
Numerical Methods for Engineers and Scientists Using MATLAB by Ramin S. Esfandiari, CRC Press, 2nd Edition, 2017
Computational Fluid Dynamics by Klasus A. Hoffmann and Steve T. Chiang, Fourth Edition, 2000.

Reference Books
Numerical Analysis By Richard L. Burden, J. Douglas Faires and Annette M. Burden, 10 E, Cengage Learning, 2016
Applied Numerical Analysis by Curtis F. Gerald and Patrick O. Wheatley, 7th Edition, Publisher: Pearson, 2003.
Theory and Application of Numerical Analysis by G. M. M. Phillips and Peter J. Taylor, 2nd edition, Academic Press, 1996
Numerical Analysis by David Kincaid and Ward Cheney, 7th Edition, Cengage Learning, 2012

Weekly Breakdown		
Week	**Sections**	**Topic**
1	3.2, 3.3, 3.4, 4.7.2	• Review of Root Finding Methods Bisection method, Regula Falsi Method (Method of False Position), Fixed-Point Method • Fixed-Point Iteration Method for a System of Nonlinear Equations
2	3.5, 3.6, 4.7.1	• Review of Secant Method • Review of Newton's Method (Newton–Raphson Method), • Newton's Method for a System of Nonlinear Equations
3	4.3.4, 4.5, 4.6	• Tridiagonal Systems: Thomas Method • Iterative Solution of Linear Systems: _Jacobi Iteration Method, _Gauss–Seidel Iteration Method • ill-Conditioning and Error Analysis: _Condition Number, Condition Number,_Ill-Conditioning, Computational Error
4	5.5.1 5.5.3 5.5.5 5.6.1 5.6.2	• Review of Interpolation Polynomial Interpolation: _ Lagrange Interpolating Polynomials, Newton Divided-Difference Interpolating Polynomials, Newton Forward-Difference Interpolating Polynomials, • Spline Interpolation Linear Splines, Quadratic Splines,
5	6.2.1 6.2.2 7.1 7.2	• Numerical Differentiation • Finite-Difference Formulas for Numerical Differentiation: Finite-Difference Formulas for the First Derivative and Second Derivative • Introduction to IVP • One-Step Methods
6	7.3	• Euler's Method: Error Analysis for Euler's Method Local And Global Truncation Errors Higher-Order Taylor Methods
7	7.4.1.1 7.4.1.2 7.4.1.3 7.4.1.4	• Runge–Kutta Method • Second-Order Runge–Kutta (RK2) Methods: Improved Euler's Method, Heun's Method Ralston's Method Graphical Representation of Heun's Method
8	7.4.2.1 7.4.2.2	• Third-Order Runge–Kutta (RK3) Methods: The Classical RK3 Method, Heun's RK3 Method
9	**Mid Semester Exam**	
10	7.4.3.1 7.4.3.2 7.4.5	• Fourth-Order Runge–Kutta (RK4) Methods: The Classical RK4 Method, Higher-Order Runge–Kutta Methods • Runge–Kutta Fehlberg (RKF) Method
11	7.6.2.1 7.6.2.2 7.6.2.3	• Numerical Solution of a System of First-Order ODEs Euler's Method for System, Heun's Method for System, Classical RK4 Method for Systems

12	7.7.1 7.7.2	• Stability: Euler's Method Euler's Implicit Method
13	8.4	• Shooting Method: Linear BVP Nonlinear BVP
14	8.5	• Finite-Difference Method: Finite-Difference Method, Linear BVP With Mixed Boundary Conditions
15	8.6 8.6.1	• MATLAB Built-In Function bvp4c for Boundary-Value Problems • Second-Order BVP
16	2.6 (Hofmann) CFD	• Finite difference Approximation for Mixed Partial Derivatives • Taylor series expansion
17	10.3.1 10.3.2	• Numerical Solution of Partial Differential Equations • Parabolic Partial Differential Equations • Crank–Nicolson (CN) Method
18	**End Semester Exam**	

MATH-807 Commutative Algebra

Credit Hours: 3-0
Prerequisite: Students must know the basic concepts of rings, quotient rings, integral domains and fields. Those students who have done Algebra / or equivalent will be preferred.

Course Objectives: This course aims to introduce students to the concepts of modules over commutative rings, Noetherian rings and modules, Artinian rings and valuation rings.

Detailed Course Contents: Rings, ideals, operations on ideals, radical of an ideal, nilradical, Jacobson radical, ideal quo one hour testtient, local rings, prime avoidance lemma, modules, module over commutative rings, submodules, operations on submodules, finitely generated modules, free module, annihilator of a of an element of a module, cyclic modules, quotient modules, direct sum and product of modules, module homomorphisms, short exact sequences, tensor product of modules, rings and modules of fractions, extended and contracted ideals in rings of fractions, Integral dependence, the going-up theorem, valuation rings, chain conditions, Noetherian rings and modules, Nakayama's lemma, primary decomposition, primary decomposition in Noetherian rings.

Learning Outcomes: On successful completion of this course, students will know Rings, ideals, operations on ideals, radical of an ideal, nilradical, Jacobson radical, ideal quotient, local rings, modules, submodules, operations on submodules, finitely generated modules, freemodules, direct sum and product of modules, module homomorphisms, isomorphism theorems of modules, tensor product of modules, rings and modules of fractions, Integral dependence, valuation rings, primary decomposition Noetherian rings and modules.

Recommended Books

1) M. F. Atiyah, and I. G. Macdonald, Introduction to Commutative Algebra, Addison- Wesley, 1994. ISBN: 9780201407518.
2) D. Eisenbud, "Commutative Algebra with a View Toward Algebraic Geometry", Springer, New York, 1995.
3) Thomas W. Hungerford, Algebra, Springer-Verlag, New York Inc. 1974.
4) David S Dummit, Richard M. Foote, Abstract Algebra, (3rd Ed.), 2004, John Wiley & Sons.

Weekly Breakdown	
Week	***Topics***
1	Commutative rings, integral domains, Euclidean domains, the greatest common divisor of two elements of a ring, and related theorems.
2	PID’s, UFD’s, and related theorems, properties of the polynomial rings, polynomial rings over fields.
3	Existence of maximal ideals of a commutative ring with unity, local rings.
4	Nilradical, Jacobson radical, related theorems, operations on ideals.
5	Radical of an ideal, ideal quotient, comaximal ideals, the Chinese Remainder Theorem.
6	Monomial ideals, operations on monomial ideals, radical of a monomial ideal, colon ideal of two monomial ideals.
7	Module over commutative rings, examples, submodules, operations on submodules.
8	Finitely generated modules, cyclic modules, Nakayama's lemma, free modules, torsion modules, torsion free modules.
9	**Mid Semester Exam**
10	Quotient modules, module homomorphisms, isomorphism theorems of modules. Direct sum and direct product of modules,
11	short exact sequences, tensor product of modules.
12	Rings and modules of fractions, localization.
13	Primary decomposition.
14	Chain conditions, Noetherian rings, and modules
15	Artinian rings and modules.
16	Primary decomposition in Noetherian rings.
17	Review
18	**End Semester Exam**

MATH-817 Advanced Functional Analysis

Credit Hours: 3-0
Prerequisite: None

Course Objectives: This course presents functional analysis from a more advanced perspective. The main objective are to 1) understand the classic results of Functional Analysisincluding Zorn's Lemma and Hahn-Banach Theorem, 2) understand the basic concepts of Fixed Point Theory, 3) know and understand the topics on approximation theory.

Previous Knowledge: A student who wishes to opt this course is recommended to have a previous knowledge of elementary functional analysis including Metric Spaces, Normed Spaces, Banach Spaces, Inner Product Spaces and Hilbert spaces. Furthermore, student is required to have a good command on elementary linear algebra.

Core Contents: Fundamental Theorems for Normed and Banach Spaces, Banach Fixed PointTheorem and its applications, Applications of Banach Fixed Point Theorem, Approximation Theory.

Detailed Course Contents: Fundamental Theorems for Normed and Banach Spaces: Zorn's Lemma, Hahn-Banach Theorem, Hahn-Banach Theorem for Complex Vector Spaces and Normed Spaces, Adjoint Operator, Reflexive Spaces, Category Theorem, Uniform Boundedness Theorem, Strong and Weak Convergence, Convergence of Sequences of Operators and Functionals, Open Mapping Theorem, Closed Linear Operators. Closed Graph Theorem.
Further Applications: Banach Fixed Point Theorem: Banach Fixed Point Theorem, Application of Banach's Theorem to Linear Equations, Applications of Banach's Theorem to Differential Equations, Application of Banach's Theorem to Integral Equations.
Approximation Theory: Approximation in Normed Spaces, Uniqueness, Strict Convexity, Uniform Approximation, Chebyshev Polynomials, Approximation in Hilbert Space

Course Outcomes: This course is specially designed for students who want to choose functional analysis and fixed point theory as their specialty. On successful completion of this course, the students will:
Be able to work with fundamental concepts in functional analysis.
Have a grasp of formal definitions and rigorous proofs in functional analysis.
Be able to apply abstract ideas to concrete problems in analysis.
Be aware of applications of basic techniques and theorems of functional analysis in other areas of mathematics, such as fixed point theory, approximation theory, and the theory of ordinary differential equations.

Text Book: Erwin Kreyszig, Introductory Functional Analysis with Applications, Wiley; First edition 1989.
Reference Books:
J. B. Conway. A Course in Functional Analysis. Springer-Verlag , New York, 1985.
George Bachman, Lawrence Narici, Functional Analysis, Dover Publications; 2nd edition, 1998.

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1, 1.3, 1.4, 1.6	**Review:** Metric spaces, Open set, Closed set, Cauchy sequence, Complete metric spaces,
2	2.2,3.1,3.2	**Review:** Normed spaces, Banach spaces, Inner product spaces, Hilbert spaces
3	4.1,4.2	Zorn's Lemma, Hahn-Banach Theorem
4	4.3, 4.5	Hahn- Banach Theorem for complex vector spaces and Normed Spaces, Adjoint Operator
5	4.6	Reflexive spaces
6	4.7	Category Theorem, Uniform Boundedness Theorem
7	4.8	Strong and Weak Convergence
8	**Mid Semester Exam**	
9	4.9	Convergence of sequences of Operators and functionals
10	4.12	Open Mapping Theorem
11	4.13	Closed Linear Operators, Closed Graph Theorem
12	5.1	Banach Fixed Point Theorem
13	5.2, 5.3	Applications of Banach's Theorem to Linear Equations and Differential Equations
14	5.4	Applications of Banach's Theorem to Integral Equations
15	6.1, 6.2	Approximation I Normed Spaces, Uniqueness, Strict Convexity
16	6.3	Uniform Approximation, Chebyshev Polynomial
17		Review
18	**End Semester Exam**	

MATH-818 Theory of Ordinary Differential Equations

Credit Hours: 3-0
Prerequisite: None

Course Objectives: Modern technology requires a deeper knowledge of the behavior of real physical phenomena. Mathematical models of real-world phenomenon are formulated as algebraic, differential or integral equations (or a combination of them). After the construction of equations, the study of their properties is necessary. At this stage the theory of ordinary differential equations plays a significant role. In this course we shall discuss the stability theory and phase-plane analysis of dynamical systems, bifurcation theory, Non-oscillation and oscillation theory and the existence theory of differential equations.

Detailed Course Contents: General theory of linear equations, Homogeneous Linear Equations with periodic coefficients: Floquet multipliers, Floquet Theorem, Stability of linearequations, Stability of linear equations by Lozinskii measures, Perturbations of linear equations, Lyapunov function method for autonomous equations, Lyapunov function methodfor nonautonomous equations, General theory of autonomous equations, Poincar´e– Bendixson Theorem, Periodic solutions and orbital stability, Basic concepts of bifurcation theory, One-dimensional bifurcations for scalar equations, One-dimensional bifurcations for planar systems, Hopf bifurcations for planar systems, Second-order linear equations, Self- adjoint second-order differential equation, Cauchy Function and Variation of Constants Formula, Sturm-Liouville problems, Zeros of solutions and disconjugacy, Factorizations and recessive and dominant solutions Oscillation and non-oscillation, Applications of the CMT to BVPs, Lower and upper solutions, Nagumo condition, Lipschitz condition and Picard- Lindelof Theorem, Equicontinuity and the Ascoli-Arzela Theorem, Cauchy-Peano Theorem, Extendability of solutions, Basic Convergence Theorem, Continuity of solutions with Respect to ICs, Kneser's Theorem. Differentiating solutions with respect to ICs, Maximum and minimum solutions.

Course Outcomes: Students are expected to understand topics such as stability theory, bifurcation theory, phase-plane analysis of dynamical systems, and existence theory of differential equations.

Text Books:
Theory of Differential Equations, W. G. Kelley, A. C. Peterson Springer, 2010.
Qingkai Kong, A Short Course in Ordinary Differential Equations, Springer 2014 (Referred as QK)

Reference Books:
1. Ordinary differential equations I.G.Petrovski, Dover Publications, Inc., 1973
2. Theory of ordinary differential equations, Coddington E.A. and Levinson, New York: McGraw-Hill, 1955.
3. Nonlinear Ordinary Differential Equations, D. W. Jordan and P. Smith, Oxford University Press, 2007

Weekly Breakdown		
Week	***Section***	***Topics***
1	Ch. 1	First order equations: Existence, Bifurcation, Stability
2	2.3	The Matrix Exponential Function, Putzer Algorithm, Lozinski measure
3	2.5	Homogeneous Linear Equations with periodic coefficients: Floquet multipliers, Floquet Theorem
5	3.4, 3.5	Stability of nonlinear systems, Semi-group property, Lyapunov function method for autonomous and non-autonomous equations, Linearization of nonlinear systems
4	3.1-3.3	Phase plane diagram, homoclinic orbits, Hamiltonian systems
6	3.6	Existence and nonexistence of periodic, Solutions, Poincar´e–Bendixson Theorem, Bendixson-Dulac Theorem, Li´enard's Theorem
7	3.7	Three-dimensional systems
8	5.1, 5.2 (QK)	Basic concepts of bifurcation theory, One-dimensional bifurcations for scalar equations
9	**Mid Semester Exam**	
10	5.3, 5.4 (QK)	One-dimensional bifurcations for planar systems, Hopf bifurcations for planar systems
11	5.1,5.2	Self-adjoint second-order differential equation: Basic concepts
12	5.3	Cauchy Function and variation of constants formula
13	5.4	Sturm-Liouville problems
14	5.5	Zeros of solutions and disconjugacy
15	5.6 ,5.7	Factorizations and recessive and dominant solutions, The Riccati Equation,
16	5.9	Green Function, Contraction Mapping Theorem (handouts)
17		Review
18	**End Semester Exam**	

MATH-819 Analysis of Fractional Differential Equations

Credit Hours: 3-0
Prerequisite: None

Objectives and Goals: The aim of the course is to motivate students to study different topicsof the theory of fractional calculus and fractional differential equations.

Core Contents: BVPs for Nonlinear Second-Order ODEs, Existence and Uniqueness Theorems, Riemann–Liouville Differential and Integral Operators, Riemann–Liouville Differential and Integral Operators, Caputo's Approach, Mittag-Leffler Functions, Existence and Uniqueness Results for Riemann–Liouville and Caputo Fractional Differential Equations

Detailed Course Contents: BVPs for Nonlinear Second-Order ODEs: Contraction Mapping Theorem, Application of the Contraction Mapping Theorem to a Forced Equation Application of Contraction Mapping Theorem to BVPs, Lower and Upper Solutions,Nagumo Condition.
Existence and Uniqueness Theorems: Basic Results, Lipschitz Condition and Picard-Lindelof Theorem, Equicontinuity and the Ascoli-Arzela Theorem, Cauchy-Peano Theorem, Extendability of Solutions, Basic Convergence Theorem, Continuity of Solutions with Respect to ICs, Kneser's Theorem, Differentiating Solutions with Respect to ICs, Maximum and Minimum Solutions.
Introduction to Fractional Calculus: Motivation, The Basic Idea, An Example Application of Fractional Calculus. Riemann–Liouville Differential and Integral Operators: Riemann–Liouville Integrals, Riemann–Liouville Derivatives, Relations Between Riemann–Liouville Integrals and Derivatives, Grunwald–Letnikov Operators. Caputo's Approach: Definition and Basic Properties, Nonclassical Representations of Caputo Operators. Mittag-Leffler Functions: Definition and Basic Properties.
Existence and Uniqueness Results for Riemann–Liouville Fractional Differential Equations. Single-Term Caputo Fractional Differential Equations: Existence of Solutions, Uniqueness of Solutions, Influence of Perturbed Data, Smoothness of the Solutions, Boundary Value Problems. Advanced Results for Special Cases: Initial Value Problems for Linear Equations,Boundary Value Problems for Linear Equations, Stability of Fractional DifferentialEquations.

Course Outcomes: Students are expected to understand:
- Existence theory for second order ordinary differential equations
- Properties of fractional operators
- Existence theory of fractional differential equations

Text Books:

1. Walter G. Kelley, Allan C. Peterson, Theory of Differential Equations, Second Edition, Springer, (2010) (Referred as KP).
2. Kai Diethelm, The Analysis of Fractional Differential Equations, Springer, (2010)(Referred as KD).

Reference Books:

1. Podlubny, Fractional Differential Equations. Academic Press, San Diego (1999).
2. R. Hilfer, Applications of Fractional Calculus in Physics, World Scientific Publishing (2000).
3. A.A. Kilbas, H.M. Srivastava, J.J. Trujillo, Theory and applications of fractional differential equations, vol 204. North-Holland mathematics studies. Elsevier, Amsterdam (2006).

Weekly Breakdown		
Week	***Section***	***Topics***
1	7.1,7.2 (KP)	BVPs for Nonlinear Second-Order ODEs: Contraction Mapping Theorem, Application of the Contraction Mapping Theorem to a Forced Equation.
2	7.3-7.5	Application of Contraction Mapping Theorem to BVPs, Lower and Upper Solutions, Nagumo Condition.
3	8.1-8.3	Existence and Uniqueness Theorems: Basic Results, Lipschitz Condition and Picard-Lindelof Theorem, Equicontinuity and the Ascoli-Arzela Theorem.
4	8.4-8.6	Cauchy-Peano Theorem, Extendability of Solutions, Basic Convergence Theorem
5	8.7-8.10	Continuity of Solutions with Respect to ICs, Kneser's Theorem, Differentiating Solutions with Respect to ICs, Maximum and Minimum Solutions.
6	1.1-1.3 (KD)	Introduction to Fractional Calculus: Motivation, The Basic Idea, An Example Application of Fractional Calculus.
7	2.1, 2.2	Riemann–Liouville Differential and Integral Operators: Riemann–Liouville Integrals, Riemann–Liouville Derivatives.
8	2.3, 2.4	Relations Between Riemann–Liouville Integrals and Derivatives, Grünwald–Letnikov Operators.
9	**Mid Semester Exam**	
10	3.1, 3.2	Caputo's Approach: Definition and Basic Properties, Nonclassical Representations of Caputo Operators.
11	4	Mittag-Leffler Functions: Definition and Basic Properties.
12	5	Existence and Uniqueness Results for Riemann–Liouville Fractional Differential Equations.
13	6.1, 6.2	Single-Term Caputo Fractional Differential Equations: Basic Theory and Fundamental Results: Existence of Solutions, Uniqueness of Solutions.
14	6.3, 6.4	Influence of Perturbed Data, Smoothness of the Solutions
15	6.5	Boundary Value Problems.
16	7.1-7.3	Advanced Results for Special Cases: Initial Value Problems for Linear Equations, Boundary Value Problems for Linear Equations, Stability of Fractional Differential Equations.
17		Review
18	**End Semester Exam**	

MATH-820 Calculus of Variations and Optimal Control

Credit Hours: 3-0
Prerequisite: None

Course Objectives: The major purpose of this course is to present theoretical ideas and analytic and numerical methods to enable the students to understand and efficiently solve optimization problems.

Core Contents: The Finite dimensional problem: The free problem. Equality constrained problem. The inequality constrained problem, Newton's Method. The basic theory of the calculus of variations: Introduction, Some examples. Critical point conditions. Additional necessary conditions. Miscellaneous results. Sufficiency theory. Several dependent variables.Optimal control, The minimal time problem, Unconstrained Reformulations. Constrained calculus of variations problems. Kuhn-Tucker reformulation. Numerical methods and results.Kuhn-Tucker method. Introduction to fractional calculus. Fractional calculus of variations, Fractional Euler–Lagrange equations

Course Outcomes: Students are expected to understand:

- The theory of the calculus of variations.
- The optimal control problems.
- Numerical methods and results for optimization.
- Fractional calculus of variations.

Text Book:

1. John Gregory, Cantian Lin, Constrained Optimization in the Calculus of Variations and Optimal Control Theory, Springer (1992).
2. Ricardo Almeida, Dina Tavares Delfim F. M. Torres, (RAD) The Variable-Order Fractional Calculus of Variations, Springer 2019.

Reference Books:

1. M. D. Intriligator, Mathematical Optimization and Economic Theory, Siam (2002).
2. Pablo Pedregal, Optimization and Approximation, Springer (2017)
3. Daniel Liberzon, Calculus of Variations and Optimal Control Theory, PRINCETON UNIVERSITY PRESS, (2012).

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1,1.2	The Finite dimensional problem: The free problem, The equality constrained problem.
2	1.3, 1.4	The inequality constrained problem, Newton's Method.
3	2.1-2.3	The basic theory of the calculus of variations: Introduction, Some examples
4	2.3	Critical point conditions.
5	2.4, 3.1	Additional necessary conditions, Miscellaneous results
6	3.2	Sufficiency theory.
7	3.3	Several dependent variables.
8	4.1	Optimal control: A basic problem
9	**Mid Semester Exam**	
10	4.2, 5.1	The minimal time problem: An example of abnormality. Unconstrained Reformulations: The optimal control problems.
11	5.2,5.3	Constrained calculus of variations problems, Kuhn-Tucker reformulation
12	6.1	Numerical methods and results: The basic Problem in calculus of variations
13	6.2	Numerical transversality conditions for general problems
14	6.3	Kuhn-Tucker method
15	2.1,2.2 (RAD)	Introduction to fractional calculus
16	3.2	Fractional calculus of variations, Fractional Euler–Lagrange equations
17		Review
18	**End Semester Exam**	

MATH-821 Analytical Approximate Solutions of ODEs

Credit Hours: 3-0
Prerequisite: None

Course Objectives: The objective of this course is to introduce analytical and approximate methods for differential equations and make students familiar with advanced topics in spectral methods.

Core Contents: The variational iteration method, The Adomian decomposition method,Perturbation method, Hamiltonian approach, Homotopy analysis method, spectral methods, Fourier and Chebyshev Series, Discrete least square approximation, Chebyshev interpolation, Tau-spectral method. Collocation spectral methods.

Detailed Course Contents: The variational iteration method: Application of the variational iteration method. The Adomian decomposition method: Application of the Adomian decomposition method. Perturbation method: Theoretical background, application of the perturbation method. Energy balance method: Theoretical background, application of the energy balance method. Hamiltonian approach: Theoretical background, application of the Hamiltonian approach. Homotopy analysis method: Theoretical background. Homotopy analysis method: application of the homotopy analysis method.
Fourier and Chebyshev Series, The trigonometric Fourier series. The Chebyshev series.Discrete least square approximation. Chebyshev discrete least square approximation. Orthogonal polynomials least square approximation. Orthogonal polynomials and Gauss-type quadrature formulas. Chebyshev projection. Chebyshev interpolation. Collocation derivative operator. General formulation for linear problems. Tau-spectral method. Collocation spectral methods: A class of nonlinear boundary value problems. Spectral-Galerkinmethods.

Learning Outcomes: On successful completion of this course students will be able to:
- Understand and apply approximate methods such as the variational iteration method,
- The Adomian decomposition method, Perturbation method, Hamiltonian approach, Homotopy analysis method
- Understand and apply spectral methods for solving differential equations.

Textbooks:

1. M. Hermann, M. Saravi, (HS) Nonlinear Ordinary Differential Equations, Analytical Approximations and Numerical Methods, Springer (2016)
2. C. I. Gheorghiu, (CIG) Spectral Methods for Differential Problems, TiberiuPopoviciu Institute of Numerical Analysis (2007)

Reference Book:

1. C. Canuto, M. Y. Hussaini, A. Quarteroni and T. A. Zang, Spectral Methods: Fundamentals in Single Domains, Springer (2006)
2. Lloyd N. Trefethen, Approximation Theory and Approximation Practice, Siam (2013).

Weekly Breakdown		
Week	***Section***	***Topics***
1	HS 2.1-2.3	The variational iteration method, application of the variational iteration method.
2	2.4, 2.5	The Adomian decomposition method, application of the Adomian decomposition method.
3	3.1	Perturbation method: theoretical background, application of perturbation method.
4	3.2	Energy balance method: theoretical background, application of energy balance method.
5	3.3	Hamiltonian approach: theoretical background, application of the Hamiltonian approach.
6	3.4	Homotopy analysis method: theoretical background.
7	3.4 (cont.)	Homotopy analysis method: application of the homotopy analysis method.
8	1.1,1.2. 1,1.2.2	General properties, Fourier and Chebyshev Series, The trigonometric Fourier series, The Chebyshev series.
9	**Mid Semester Exam**	
10	1.2.3	Discrete least square approximation.
11	1.2.4,1. 2.6	Chebyshev discrete least square approximation, Orthogonal polynomials least square approximation, Orthogonal polynomials and Gauss-type quadrature formulas
12	1.3,1.4	Chebyshev projection, Chebyshev interpolation.
13	1.4 (cont.)2.1	Chebyshev interpolation (cont.) Collocation derivative operator. The idea behind the spectral methods.
14	2.2,2.3	General formulation for linear problems, Tau-spectral method.
15	2.4	Collocation spectral methods (pseudo spectral), A class of nonlinear boundary value problems.
16	2.5	Spectral-Galerkin methods.
17		Review
18	**End Semester Exam**	

MATH-822 Mathematical Modelling-I

Credit Hours: 3-0
Prerequisite: None

Course Objectives: The course focuses on the application of "dimensional methods" tofacilitate the design and testing of engineering problems. It aims to develop a practical approach to modeling and dimensional analysis. This course will be well received and will prove to be an invaluable reference to researchers and students with an interest dimensional analysis and modeling and those who are engaged in design, testing and performancesevaluation of engineering and physical system.

Core Contents: The course includes the theory of matrix algebra and linear algebra, thetheory of dimension, transformation of dimensions and structure of physical variables, dimensional similarities and models law. This course will cover the nature of dimensional analysis use in mathematical modeling.

Detailed Course Contents: Mathematical Preliminaries, Matrices and Determinants, Operation with Matrices, The rank of matrices and Systems of linear equations, Formats and Classification, Numerical, Symbolic and Mixed format, Classification of Physical Quantities, dimensional system, General Statement, The SI system, Structure, Fundamental dimension, Derived dimensional units with and without specific names, Rules of etiquettes in Writing dimensions.
Other than SI dimensional systems, A note on the classification of dimensional systems, Transformation of Dimensions, Numerical equivalences, Techniques, Examples, Problems, Arithmetic of Dimensions, Dimensional Homogeneity. Equations, graphs, Problems, Structure of Physical Relations, the dimensional matrix, Number of independent sets of products of given dimension 1,11, Special case, Buckingham's theorem, Selectable and non selectable dimensions, Minimum number of independent product of variables of given dimension, Constancy of the sole dimensionless product, Number of dimension equals or exceeds the number of variables, Systematic determination of Complete Set of Products of Variable Transformations, Theorems related to some specific transformations, Transformations between systems of different d matrices, Number of Sets of Dimensionless Products of Variables, Distinct and equivalent sets, Changes in dimensional set not affecting the dimensional variables, Prohibited changes in dimensional set. Relevancy of Variables, Dimensional irrelevancy, Condition, Adding a dimensionally irrelevant variables to a set of relevant variables, Physical irrelevancy, Problems, Economy of Graphical Presentation, Number of curves and charts, Problems, Forms of Dimensionless Relations, General classification, Monomial is Mandatory, Monomial is impossible, Reconstructions, Sequence of Variables in the Dimensional Set, Dimensionless physical variable is present, Physical variables of identical dimensions are present, Independent and dependent variables.

Learning Outcomes: Students are expected to understand Fundamentals dimension of dimensional analysis.
Text Books:

1. Thomas Szitres, Applied Dimensional Analysis and Modeling, Elsevier Inc., 2007. (Referred as TS).
2. S.H. Friedberg, A.J. Insel, L.E.Spence, Linear Algebra, Prentice-Hall, Inc., Englewood Cliffs, N.J. USA,979 (referred as FIS)

Weekly Breakdown		
Week	***Section***	***Topics***
1	TS, Ch. 1, FIS, Ch. 3	Mathematical Preliminaries, Matrices and Determinants, Operation with Matrices, The rank of matrices and Systems of linear equations
2	TS Chs. 2, 3	Formats and Classification, Numerical, Symbolic and Mixed format Classification of Physical Quantities, dimensional system, General Statement, The SIsystem
3	Ch 3	Structure, Fundamental dimension, Derived dimensional units with and withoutspecific names, Rules of etiquettes in Writing dimensions Other than SI dimensional systems
4	Chs 3,4	A note on the classification of dimensional systems, Transformation of Dimensions,Numerical equivalences, Techniques, Examples, Problems
5	Chs 5, 6	Arithmetic of Dimensions, Dimensional Homogeneity
6	Chs 6,, 7	Equations, graphs, Problems, Structure of Physical Relations, the dimensional matrix, Number of independent sets of products of given dimension 1,11, Special case
7	Ch 7	Buckingham's theorem, Selectable and non selectable dimensions, Minimum number of independent product of variables of given dimension, Constancyof the sole dimensionless product
8	Chs 7,8	Number of dimension equals or exceeds the number of variables Systematic determination of Complete Set of Products of Variable
9	**Mid Semester Exam**	
10	Ch 9	Transformations, Theorems related to some specific transformations, Transformations between systems of different d matrices
11	Ch 10	Number of Sets of Dimensionless Products of Variables Distinct and equivalent sets, Changes in dimensional set not affecting the dimensionalvariables, Prohibited changes in dimensional set
12	Ch 11	Relevancy of Variables, Dimensional irrelevancy, Condition, Adding a dimensionally irrelevant variables to a set of relevant variables,
13	Chs 11, 12	Physical irrelevancy, Problems, Economy of Graphical Presentation Number of curves and charts, Problems
14	Ch 13	Forms of Dimensionless Relations, General classification, Monomial is Mandatory,Monomial is impossible, Reconstructions
15	Ch 14	Sequence of Variables in the Dimensional Set, Dimensionless physical variable is present,
16	Ch 14	Physical variables of identical dimensions are present, Independent and dependent variables
17		Review of Material
18	**End Semester Exam**	

MATH-XXX Advanced Topology

Credit Hours: 3-0
Prerequisite: Nil

Course Objectives: The course aims at developing an understanding about advanced concepts of Topology which are the basic tools of working mathematicians in a variety of fields. It covers some cover concepts including compactness and connectedness and explains how these concepts of Analysis are generalized to Topology.

Core Contents: Topological Spaces, Neighborhood, Bases, Initial & Final Topology, Quotient Spaces, Inadequacy of Sequences, Nets, Filters & Ultra Filters, Lindelöf Spaces, Separable Spaces, Compactness, Compactness in terms of filters, Locally Compact Spaces, One-point compactification, Stone-Cech Compactification, Para-compactness, Connectedness, Connected Components, Pathwise & Locally Connected Spaces,

Detailed Course Contents: Topological Spaces, Neighborhood, Neighborhood base, Subbases, Local Bases, Bases, Initial/Weak Topology and its Applications, Final/Strong Topology and its Applications, Quotient Spaces, Inadequacy of Sequences, Nets and their properties, Filters, Filter bases, Ultra Filters, Topology induced by filters, Relation b/w filters & Nets, Lindelöf Spaces, Separable Spaces, Compactness, Compactness in terms of Closedness & filters, Countable compactness, Limit-point compactness, One-point compactification, Stone-Cech compactifications, Connectedness, Connected components, Totally Disconnected spaces, Locally connected spaces and its applications, Pathwise connectedness and its relation to connectedness.

Course Outcomes: Upon completion of this course, the student should be able to:

- Continuous mappings, Disjoint Homeomorphism, Weak and Strong topologies, Quotient spaces
- Inadequacy of Sequences, Nets, Filters & Ultra Filters
- Lindelöf Spaces, Separable Spaces
- Compactness, Countable, Limit-point and local compactness
- One-point & Stone-Cech Compactifications
- Connectedness, Connected components, Totally disconnectedness, Pathwise & Local Connectedness

Text Book: S. Willard, “General Topology”, Dover Publications; Illustrated Edition, (2004)

Reference Books:

1. James R. Munkres, “Topology”, Prentice, Hall, Inc., 2nd Edition (2000)
2. T. D. Bradley, T. Bryson, J. Terilla, “Topology: A Categorical Approach”, MIT Press, (2020)
3. G. Preuss, “Foundations of Topology: An Approach to Convenient Topology”, Springer, 2nd Edition, (2002).
4. J. Kelly, “General Topology”, Springer, (2005).

Weekly Breakdown		
Week	***Section***	***Topics***
1	**Sec. 3-4**	Review of Topological spaces and Examples, Neighborhood operators, Topology induced by neighborhoods, Neighborhood bases, Open, closed, interiors and closures in terms of neighborhoods
2	**Sec. 5-6**	Subbases, Bases, Local bases and their properties, Subspaces and its properties, and related results
3	**Sec. 7**	Continuous functions between topologies, and their characterizations using neighborhood operators, characterizations of spaces using continuous mappings, Continuous functions to and from a plane., Disjoint homeomorphisms
4	**Sec.8**	Weak Topologies and their applications, Box products and their related results, Tychonoff Topologies
5	**Sec. 9**	Strong/Final Topologies and their applications, Quotient spaces, identification spaces, Quotients vs Decompositions
6	**Sec. 10**	Inadequacy of sequences, sequentially convergences, 1st, and 2nd countable and its applications
7	**Sec. 11**	Nets, Ultra nets and their examples, subnets and related results, Net convergence in topologies
8	**Sec. 12**	Filters, Ultrafilters, Topologies induced by filters, Filter convergence in topological spaces, Relationship between filters and nets
9	**Mid Semester Exam**	
10	Sec.13-14	Lower Separation axioms and related results, Regular and completely regular spaces
11	Sec. 15-16	Normal spaces and related results, Urysohn Lemma and Tietze Extension Theorem, Shrinkable spaces, Separable and Lindelöf spaces and Results
12	Sec. 17	Compactness, Compactness in terms of neighborhoods and filters, sequentially compactness and their related results, Countable compactness, and related theorems
13	Sec. 18	Locally compact spaces, examples and its relations with compactness, countable compactness and sequentially compactness, and their related results
14	Sec. 19	Compactifications, Alexandroff Compactifications, Stone-Cech Compactifications
15	Sec. 26	Connectedness and examples, Connectedness in terms of neighborhood and filters, Mutual Separated spaces, Connected components and their related results
16	Sec. 27	Pathwise connectedness and locally connectedness, examples and their related results and their relation with connectedness and mutual separateness
17	Sec. 29	Totally disconnected spaces, examples and related results, Zero-dimensional spaces, examples, and related theorems.
18	**End Semester Exam**	

MATH-903 Partial Differential Equations-I

Credit hours: 3-0
Prerequisite: None

Course Objectives: Modern technology requires a deeper knowledge of the behavior of real physical phenomena. Mathematical models of real world phenomenon are formulated as algebraic, differential or integral equations (or a combination of them). After the construction of equations the study of their properties is necessary. At this stage the theory of ordinary differential equations plays a significant role. Partial Differential Equations (PDEs) are at the heart of applied mathematics and many other scientific disciplines. PDEs are at the heart of many scientific advances. The behavior of many material object in nature, with time scales ranging from picoseconds to millennia and length scales ranging from sub-atomic to astronomical, can be modelled by PDEs or by equations with similar features. Indeed, many subjects revolve entirely around their underlying PDEs. The role of PDEs within mathematicsand in other sciences is fundamental and is becoming increasingly significant. At the same time, the demands of applications have led to important developments in the analysis of PDEs, which have in turn proved valuable for further different applications. The goal of the course is to provide an understanding of, and methods of solution for, the most important types of partial differential equations that arise in Mathematical Physics. Advanced topics such as week solutions and discontinuous solutions of nonlinear conservation laws are also considered.

Detailed Course Contents: First-order Partial Differential Equations: Linear First-order Equations, The Cauchy Problem for First-order Quasi-linear Equations,Fully-nonlinear First- order Equations, General Solutions of Quasi-linear Equations. Second-order Partial Differential Equations: Classification and Canonical Forms of Equations in Two Independent Variables, Classification of Almost-linear Equations in R^n . One Dimensional Wave Equation: The Wave Equation on the Whole Line. D' Alembert Formula, The Wave Equation on the Half-line, Reflection Method. Mixed Problem for the Wave Equation, Inhomogeneous Wave Equation, and Conservation of the Energy. One Dimensional Diffusion Equation: The Diffusion Equation on the Whole Line, Diffusion on the Half-line, Inhomogeneous Diffusion Equation on the Whole Line, Maximum- minimum Principle for the Diffusion Equation. Weak Solutions, Shock Waves and Conservation Laws: Weak Derivatives and Weak Solutions Conservation Laws, Burgers' Equation, Weak Solutions. Riemann Problem, Discontinuous Solutions of Conservation Laws, Rankine-Hugoniot Condition.
The Laplace Equation: Harmonic Functions. Maximum-minimum Principle, Green's Identities, Green's Functions, Green's Functions for a Half-space and Sphere, Harnack's Inequalities and Theorems.

Course Outcomes: Students are expected to understand topics such as first and second orderlinear classical PDEs as well as nonlinear equations. Explicate formulae and derive propertiesof solutions for problems with homogenous and inhomogeneous equations; without boundaries and with boundaries.

Textbooks: Ioannis P Stavroulakis, Stepan A Tersian, Partial Differential Equations: An Introduction with Mathematica and Maple, World Scientific, 2004.

Reference Books:

1. Walter A Strauss, Partial Differential Equations: An introduction, John Wiley & Sons, 2008.

2. Peter J. Olver, Introduction to Partial Differential Equations, Springer, 2014.

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1-1.2	Introduction to partial differential equations, Linear First-order Equations.
2	1.3	The Cauchy Problem for First-order Quasi-linear Equations. Existence and blowup of solution.
3	1.4	Quasi-linear Equations: theory and methods of general solution.
4	Handouts	Classification of system of partial differential equations. Method of solutions for system of partial differential equations.
5	1.5	Fully-nonlinear First-order Equations: Theory and methods of solution.
6	2.1, 2.2	Methods of solution for Linear Equations. Classification and Canonical Forms of Equations in two Independent Variables.
7	3.1, 3.2	The Wave Equation on the Whole Line. D'Alembert Solution, The Wave Equation on the Half-line.
8	3.3	Reflection Method, Mixed Problem for the Wave Equation.
9	**Mid Semester Exam**	
10	3.4	Inhomogeneous Wave Equation.
11	3.5	Conservation of the Energy.
12	4.1	Maximum-minimum Principle for the Diffusion Equation
13	4.2	The Diffusion Equation on the Whole Line.
14	4.3, 4.4	Diffusion on the Half-line. Inhomogeneous Diffusion Equation on the Whole Line.
15	5.1,5.2	Weak Derivatives and Weak Solutions, Conservation Laws.
16	5.3,5.4	Burgers' Equation, Weak Solutions. Riemann Problem.
17		Review
18	**End Semester Exam**	

MATH-905 Symmetry Methods for Differential Equations-I

Credit Hours: 3-0
Prerequisites: None

Course Objectives: This lecture course aims to introduce students to the basic concepts of Symmetry Methods. Whereas there are standard techniques for solving differential equations, apart from the first order equations there are no standard techniques for solving non-linear differential equations. Lie had developed an approach to try to determine substitutions, whichcould be used to reduce the order of an ODE, or the number of independent variables of a PDE. This field has made dramatic advances under the name of "symmetry analysis". In this course Lie groups, local Lie groups and Lie algebras will be reviewed. Then the symmetries of algebraic and differential equations will be discussed. Next the techniques for finding the symmetries of an ODE, and their use for solving it will be presented. This will be extended tosystems of ODEs. The technique of finding differential invariants will be discussed with reference to some particular examples. The symmetries of PDEs will also be discussed and some examples presented.

Core Contents: Lie groups, local Lie groups and Lie algebras. Symmetries of algebraic and differential equations. Techniques for finding the symmetries of an ODE and their use for solving it. Extension to systems of ODEs. Differential invariants. The symmetries of PDEs. Techniques for finding the symmetries of a PDE, and their use for reducing the number of independent variables.

Detailed Curse Contents: One-parameter group of point transformations and their generators, Transformation laws, Extensions of transformations. Generators of point transformations and their prolongation; first formulation of symmetries; ODEs and PDEs of 1st order, Second formulation of symmetries Lie symmetries of 1st and 2nd order ODEs. Lie symmetries of 2nd order ODEs; higher order ODEs and linear nth order ODEs. The use of symmetries to solve 1st order ODEs. Lie algebras for infinitesimal generators. Examples of Lie Algebras. Subgroups and subalgabras; Invariants and Differential Invariants. The use of symmetries for solving 2nd order ODEs admitting a G_2. Second integration strategy. The use of symmetries for solving 2nd order ODEs admitting more than two symmetries. Higher order ODES admitting more than one Lie point symmetry. System of second order differential equations. Symmetries more general than Lie point symmetries. Symmetries of partial differential equations. Use of symmetries for solving partial differential equations of 1st order. 2nd order PDEs; Generating solutions by Symmetry transformations.

Course Outcomes: On successful completion of this course, students will be able to

- understand the basic concepts of the Lie point symmetries
- determine the symmetries of differential equations
- use symmetries to get the solutions or reduce order of ordinary differential equations
- determine the symmetries of system of ordinary differential equations
- determine the Noether symmetries of differential equations
- understand the need of contact symmetries of differential equations

Textbook: Hans Stephani, Differential Equations: Their Solution Using Symmetries, Cambridge University Press 1990

Reference book: N. H. Ibragimov, Elementary Lie Group Analysis and Ordinary Differential Equations, John Wiley and Sons1999.

Weekly Breakdown		
Week	***Section***	***Topics***
1	2.1-2.3	One-parameter group of point transformations and their generators,Transformation laws, Extensions of transformations.
2	2.4, 3.1-3.2	Generators of point transformations and their prolongation; first formulation of symmetries; ODEs and PDEs of 1st order
3	3.3-3.4, 4.1-4.2	Second formulation of symmetries Lie symmetries of 1st and 2nd order ODEs.
4	4.3, 4.4	Lie symmetries of 2nd order ODEs; higher order ODEs and linear nth order ODEs.
5	5.1-5.2	The use of symmetries to solve 1st order ODEs.
6	6.1-6.2	Lie algebras for infinitesimal generators. Examples of Lie Algebras.
7	6.3-6.5	Subgroups and subalgabras; Invariants and Differential Invariants.
8	7.1-7.2	The use of symmetries for solving 2nd order ODEs admitting a G_2.
9	**Mid Semester Exam**	
10	7.3-7.4	Second integration strategy.
11	7.5, 8.1-8.3	The use of symmetries for solving 2nd order ODEs admitting more than two symmetries.
12	9.1-9.5	Higher order ODES admitting more than one Lie point symmetry.
13	10.1-10.3	System of second order differential equations.
14	11.1-11.5	Symmetries more general than Lie point symmetries.
15	15.1-15.3 16.1	Symmetries of partial differential equations. Use of symmetries for solving partial differential equations of 1st order.
16	16.2, 17.1-17.4	2nd order PDEs; Generating solutions by Symmetry transformations.
17		Review
18	**End Semester Exam**	

MATH 908 Fixed Point Theory

Credits Hours: 3-0
Prerequisites: Some basic knowledge of Analysis

Course objectives: Aims: to teach elements of the metric fixed point theory with applications. Objectives: a successful student will:
Be acquainted with some aspects of the metric fixed point theory;
Have sufficient grounding in the subject to be able to read and understand some researchtexts;
be acquainted with the principal theorems as treated and their proofs and able to use themin the investigation of examples.

Detailed Course Contents: The course includes Lipschitzian, contraction, contractive & non-expansive mappings, Banach's contraction principal with application to differential and integral equations, Brouwer's fixed point theorem with applications, Schauder's fixed point theorem with applications, uniformly convex and strictly convex spaces, properties of non- expansive mappings, Extension's of Banach's contraction principal, Fixed Point Theory in Hausdorff Locally Convex Linear Topological Spaces, Contractive and non-expansive Multivalued maps.

Text Book:

1. Introductory Functional Analysis with Applications, E. Kreyszig, John Wiley &Sons, New York, 1978.(IFAA)
2. Fixed Point Theory and Applications, Agarwal, R., Meehan, M., &O'Regan, (Cambridge Tracts in Mathematics). Cambridge: Cambridge University Press, 2001. (FPTA)

Reference Books:

1. An Introduction to Metric Spaces and Fixed Point Theory, M. A. Khamsi, W. A. Kirk,John Wiley & Sons, New York, 2001.
2. Fixed Point Theory, V. I. Istratescu, D. Reidel Publishing Company, Holland, 1981.

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1, 1.2, 1.3 (IFAA)	Metric Spaces, Examples of metric spaces. Open sets closed sets.
2	2.2, 2.3 (IFAA)	Normed spaces, Banach spaces, Properties of normed spaces
3	5.1 (IFAA)	Banach fixed point theorem
4	5.2 (IFAA)	Applications of Banach’s Theorem to Linear equations
5	5.3 (IFAA)	Applications of Banach’s Theorem to Differential equations
6	5.4 (IFAA)	Applications of Banach’s Theorem to Integral equations
7	1 (FPTA)	Contractions
8	2(FPTA)	Non-expansive maps
9	**Mid Semester Exam**	
10	3(FPTA)	Continuation Methods for Contractive and non-expansive mappings
11	4(FPTA)	The Theorems of Brouwer , Shauder
12	5(FPTA)	Nonlinear alternatives of Leray-Shauder type
13	6(FPTA)	Continuation Principles for Condensing Maps
14	7(FPTA)	Fixed point Theorem in Conical Shells
15	8(FPTA)	Fixed Point Theory in Hausdorff Locally Convex Linear Topological Spaces
16	9(FPTA)	Contractive and non-expansive Multivalued maps
17		Review
18	**End Semester Exam**	

MATH 909 Continuum Mechanics-I

Credits: 3-0
Prerequisites: None

Course Objectives: This lecture course aims to introduce students to the basic concepts of Continuum Mechanics and linear elasticity

Core Content: Tensors, basic constitutive laws of linear elasticity, stress and strain tensors in linear elasticity, elastic materials and symmetries, elasticity and problems related to reflection, refraction of waves, surface waves and wave guides.

Detailed Contents: Tensors: Definition of a tensor of order 2 and its extension to higher orders in a recursive manner. Change of basis. Covariant and contravariant tensors. Tensor algebra.
Symmetry in elastic materials: Periodicity in crystals, lattices, unit cell. The seven crystal systems.
Effect of symmetry on tensors: Reduction of the number of independent components of a tensor due to crystal symmetry, matrices for group symmetry elements in crystals, effect of a centre of symmetry and an axis of symmetry.
Static elasticity: The strain and stress tensors, equilibrium conditions. Hooke's Law. The elasticity tensor. Elastic energy in a deformed medium. Restrictions imposed by crystal symmetry on the number of independent elastic moduli.
Dynamic elasticity: Propagation equation, properties of elastic plane waves. Propagation along directions linked to symmetry. Elastic waves in an isotropic medium.
Reflection and refraction: Reflection of an SH wave from the surface of a half space. Reflection and refraction of a P-wave and and SV wave. Mode conversion.
Surface waves: The Rayleigh wave, uniqueness of the wave speed. The Love wave.
Wave guides: The Rayleigh Lamb dispersion relation for an isotropic plate. Lamb waves in an anisotropic plate.

Learning Outcomes: On successful completion of this course, students are expected to have:

- Understood mathematical definition of a tensor of rank n as a bilinear mapping from V^{n-1} to V, where V is a vector space. He/she should be adept at tensor algebra.
- Understood the symmetry groups associated with various classes of elastic materials.
- Understood equations of motion describing the dynamics of a continuum.
- Understood wave propagation in an anisotropic material.
- Understood the theory of Rayleigh waves, Love waves and Rayleigh-Lamb waves in a wave guide.
- Understood reflection and transmission of waves across an interface.

Text books

- ED: E. Dieulesaint and D. Royer, Elastic Waves in Solids-I, Free and Guided Waves, John Wiley and Sons.(2000)
- JDA: J. D. Achenbach, Wave Propagation in Elastic Solids, North Holland.(1973)

Reference books

1. N.D. Critescu, E.M. Cracium and E. Soos, Mechanics of Elastic Components, Chapman and Hall.
2. T.C.T. Ting, Anisotropic Elasticity, Oxford University Press.

Weekly Breakdown		
Week	***Section***	***Topics***
1	Instructor's choice for book	Vector space, tensor of rank 2 as a linear mapping from V to V. Orthonormal bases.
2	-do-	Tensor of rank n. Tensor algebra.
3	ED 2.1-2.2	Symmetry in elastic materials, seven crystal systems.
4	ED 2.6	Reduction of number of independent components of a tensor due to symmetry.
5	ED 3.1	The strain and stress tensors. Physical interpretation of components. Equilibrium conditions.
6	ED 3.2	The elasticity tensor
7	ED 3.2	Restrictions imposed by crystal symmetry on the number of independent elastic moduli. Matrix representations for the seven crystal systems.
8	JDA 1.2	Linearzed theory of wave propagation, Waves in one dimensional longitudinal stress,
9	**Mid Semester Exam**	
10	JDA 2.4, 2.10	Elastic waves in an isotropic medium. The scalar and vector potentials.
11	JDA 4.1, 4.2	Plane waves, Time-harmonic plane waves
12	JDA 4.4 5.1-5.2, 5.4	Two dimensional wave motion with axial symmetry Joined half spaces
13	JDA 5.5-5.7	Reflection of an SH wave from the free surface of a half space. Reflection and transmission of a P wave and an SV wave, mode conversion.
14	JDA 5.11	The Rayleigh wave. Uniqueness of the phase speed
15	JDA 6.6	Propagation in a layer. Love wave.
16	JDA 6.7-6.8	Wave guides. The Rayleigh-Lamb dispersion relation in an isotropic plate. Analysis of the shape of the spectrum. The anomalous Lamb modes.
17		Review
18	**End semester Exam**	

MATH-911 Special Functions

Credit Hours: 3-0
Prerequisite: None

Objectives and Goals: This course deals with the theory of functions of real and complex variables. While the original definition of a function may be in a more limited domain it can often be extended to larger domains by analytic continuation. As such, integral transforms that extend the domain of applicability are needed to study the functions in themselves. We will first discuss integral transforms and "fractional calculus" and go on to the special functions used in other areas of mathematics, in Statistics and in number. We then go on to the special functions of mathematical physics that originated as solutions of 2nd order linear ordinary differential equations and their continuation by integral representations.

Core Contents: Transform Methods, Fractional Calculus, Special Functions.

Detailed Course Contents: The integral operator and integral transforms. Linear and non- linear integral transforms. Fourier transforms of classical functions and conditions for existence. Properties of Fourier transform. Convolutions properties of Fourier transform. Distributions and generalized functions. Fourier transforms of generalized functions. Poisson summation formulae and applications. The Laplace transform and conditions for its existence. Basic properties of Laplace transform. Convolutions. Inverse Laplace transforms. Differentiation and integration of Laplace transforms. Use of Laplace transforms for differential and integral equations. Fractional calculus and its applications. Fractional differential and integral equations. The Hilbert transform and its properties. Extension to the complex domain. The Steiltjes transform its properties and inversion theorems. The Mellin transform. The gamma and beta functions and their integral representations. Properties and asymptotic expansion of the gamma function. The probability integral and its properties for real and complex domains. The exponential and logarithmic integrals. Hypergeometric functions and Legendre functions. The hypergeometric series and its analytic continuation. Properties of the hypergeometric functions. Confluent hypergeometric functions. Generalizedhypergeometric functions.

Learning Outcomes: On successful completion of this course, students will be able to:

- Understand the concepts of integral transforms.
- Understand the notion of fractional calculus.
- Know the transform methods and special functions with their properties and applications.

Text books:

1. L. Debnath and D. Bhatta, Integral Transforms and Their Applications Chapman & Hall/CRC; Second Edition (October 2006)
2. N.N. Lebedev, Special Functions and their applications (tr. R.R. Silverman) Dover Publications (Revised Editions, June 1972)

Reference Books:

1. M. Ya. Antimirov, A. A. Kolyshkin and Remi Vaillancourt, Applied Integral Transforms, The American Math. Society, (1993)
2. Nikiforov and Uvarov, Special Functions of Mathematical Physics, Springer, 1988

Weekly Breakdown		
Week	***Section***	***Topics***
1	2.1-2.5, 2.9	Fourier transforms of classical functions and conditions for existence. Properties of Fourier transform. Convolutions properties of Fourier transform.
2	2.10-2.13, 3.1-3.4	Fourier transforms of generalized functions. Poisson summation formulae and applications to the solution of differential and integral equations. The Laplace transform and conditions for its existence. Basic properties of the Laplace transform.
3	3.4 – 3.7	Convolutions, Inverse Laplace transforms. Differentiation and integration of Laplace transforms.
4	5.1 – 8.4	Fractional calculus and its applications. Fractional differential and integral equations
5	6.1-6.3	Laplace transform of fractional integrals and derivatives, Mittage-Lefller function and its properties, Fractional ordinary differential equations.
6	6.4,6.5	Fractional integral equations, Initial value problems for fractional differential equations
7	8.1-8.4	Mellin Transforms: Properties and application of Mellin transforms
8	8.5-8.7	Mellin transform of fractional integrals and derivatives
9	**Mid Semester Exam**	
10	9.1-9.4	The Hilbert transform and its properties, Extension to the complex domain
11	9.7-9.8	The Steiltjes transform its properties and inversion theorems.
12	NNL 1.1 – 1.5	The gamma and beta functions and their integral representations. Properties and asymptotic expansion of the gamma function. Incomplete gamma function.
13	2.1 - 2.4	The probability integral and its properties for real and complex domains. Asymptotic representation of probability integrals.
14	3.1 - 3.4	The exponential and logarithmic integrals. Asymptotic representation of exponential integrals.
15	7.1 – 7.6	Hypergeometric functions and Legendre functions. The hypergeometric series and its analytic continuation
16	9.1 – 9.5 9.7, 9.8	Properties of the hypergeometric functions. Confluent hypergeometric functions. Generalized hypergeometric functions
17		Review
18	**End semester exam**	

MATH-941 Graph Theory

Credit Hours: 3-0
Prerequisites: None

Course Objectives: Graph theory is a stand-alone branch of pure mathematics that has links across the mathematical spectrum. The primary objective of the course is to introduce students to the beautiful and elegant theory of graphs, focusing primarily on finite graphs.
Previous Knowledge: Basic knowledge of linear algebra is needed.

Core Contents: Basics of graph theory, Path, Cycles, Trees, Matchings, Connectivity and Network Flows, Coloring, Planar graphs.

Detailed Contents: The basics of graph theory: Definition of a graph, graphs as models, matrices, isomorphism, decomposition, paths, cycles, trails, bipartite graphs, Eulerian circuits, vertex degrees and counting, directed graphs.
Trees: Properties of trees, distances in trees and graphs, spanning trees in graphs, decomposition and graceful labeling, minimum spanning trees, shortest paths, trees in computer science.
Matching: Maximum matchings, Hall's matching condition.
Connectivity: Connectivity, edge connectivity, blocks, 2-connected graphs, maximum network flow.
Coloring: Vertex coloring, chromatic number, clique number, upper bounds on chromatic number.
Planar graphs: Drawing in the plane, dual graphs, Euler's Formula.

Text Book: Douglas B. West, Introduction to Graph Theory, Second Edition, Pearson Education Inc, 2001.

Reference Books:

1. Reinhard Diestel, Graph Theory, Third edition, Springer 2005.
2. J.A. Bondy and U.S.R. Murty, Graph Theory, Springer 2010.
3. B. Bollobas, Modern Graph Theory, Springer 1998.

Weekly Breakdown	
Week	**Topics**
1	Definition of graphs: loops, multiple edges, simple graphs, neighbors. Graph as models: Complement, clique, independent set, bipartite graphs
2	Chromatic number, k-partite graphs, path, cycle, subgraphs. Matrices and Isomorphism: adjacency matrix, incidence matrix, degree of vertex
3	Isomorphism, n-cycle, complete graph, complete bipartite graphs. Decomposition: self-complementary graphs, decomposition
4	Triangle, paw, claw, kite, Petersen graph, girth. Connection in graphs: walks, trail, u,v-walk and path, internal vertices, length of walk and path.
5	Connected and disconnected graphs, components of graph, isolated vertex, cut-edge, cut-vertex, induced subgraphs, union of graphs, Eulerian graphs, Eulerian circuits, even graph
6	Vertex degrees and counting: degree of vertex, regular and k-regular graphs, neighborhood, order of a graph, Counting and bijections: degree sum formula, k-dimensional cube. Graphicsequence, introduction of directed graphs
7	Trees: acyclic graph, forest, leaf, spanning subgraphs, spanning trees, star, properties of trees
8	Distances in trees and graphs: distance, diameter, eccentricity, radius, center of a graph, Wiener index, contraction of edges, graceful labelling
9	**Mid Semester Exam**
10	Minimum spanning tree: Kruskal Algorithm, Shortest path: Dijkstra's Algorithm
11	Trees in Computer Science: Rooted tree, children, ancestors, descendants, rooted plane tree, binary tree, left child, right child
12	Matchings: matching, perfect matchings, maximum and maximal matchings, M-alternating and augmenting paths, symmetric difference, Hall's matching condition
13	Connectivity: vertex cut, connectivity and k-connected graphs, edge-connectivity, edge-connectivity and k-edge-connected graphs,
14	Network Flow Problems: Network, capacity, source and sink vertex, flow, maximum network flow, Ford-Fulkerson labeling algorithm
15	Coloring of graphs: k-coloring, proper coloring, k-colorable graphs, chromatic number, k-chromatic graphs, greedy coloring algorithm
16	Planar graphs: curve, polygonal curve, crossing, planar graphs, planner embedding, closed curve, simple curve, region, faces, dual graphs, Euler's formula
17	Review
18	**End Semester Exam**

MATH-943 Convex Analysis

Credit hours: 3-0
Prerequisites: ~~MATH-802 Analysis~~

Course Objectives: Although the systematic study of convex sets started by the end of the 19th century, convexity only became an independent branch of mathematics by the middle of the 20th century. Convexity combines conceptual tools from geometry, analysis, linear algebra and topology, and plays a crucial role in number theory, optimization, inequality theory, combinatorial geometry and game theory. The course is focused on convex sets and convex functions, showing applications to optimality theory in convex programming and conjugacy theory.

Core Contents: Basic concepts of convex analysis, Topological properties of convex functions, Duality correspondence, Representation and inequalities and Bifunctions and generalized convex program.

Detailed Course Contents: Affine sets, convex sets and cones, the Algebra of convex sets, convex functions, functional operations, relative interiors of convex sets, closures of convex functions, some closeness criteria, continuity of convex functions, separation theorems, conjugates of convex functions, support functions, polers of convex sets and functions, dual operations, Caratheodory's theorem, extreme points and faces of convex sets, polyhedral convex sets and functions, some applications of polyhedral convexity, Helly's theorem and systems of inequalities, directional derivatives and sub gradients, constrained extremum problems, saddle functions and minimax theory.

Learning Outcomes: Students are expected to understand the fundamentals of convex analysis, Topological properties of convex functions, Duality correspondence, Representationand inequalities.

Text Book: R. Tyrrel Rockafeller, Convex Analysis, Princeton University press, 1970.

Weekly Breakdown		
Week	***Section***	***Topics***
1	Part I Sec. 1,2	Affine sets, convex sets and cones
2	Part I Sec. 3,4	The Algebra of convex sets, convex functions
3	Part II Sec. 5, 6	Functional Operations, Relative interiors of convex sets
4	Part II Sec. 7, 8	Closures of convex functions, Recession cones and unboundedness
5	Part II Sec. 9, 10	Some closeness criteria, Continuity of convex functions
6	Part II Sec. 11, 12	Separation Theorems, Conjugates of convex functions
7	Part III Sec. 13, 14	Support function
8	Part III Sec. 14, 15	Polars of Convex sets, polars of convex functions
9	**Mid Semester Exam**	
10	Part III Sec. 16	Dual operations
11	Part IV Sec. 17, 18	Caratheodorey's Theorem, Extreme points and faces of convex sets
12	Part IV Sec. 19, 20	Polyhedral Convex sets and functions, Some applications of Polyhedral convexity
13	Part IV Sec. 21, 22	Helly's Theorem and systems of inequalities, Linear inequalities
14	Part V Sec. 23,24	Directional derivatives and sub gradients, Differential continuity and Monotonicity
15	Part VI Sec 27, 28	The minimum of a convex function, Ordinary convex programs and Lagrange multipliers
16	Part VI Sec 29, 30	Bifunctions and generalized convex program, Fenchel's duality theorem
17		Review
18	**End Semester Exam**	

MATH-944 Semigroup Theory of Operators

Credit Hours: 3-0
Prerequisite: None

Course Objectives: PhD/M.Phil and graduate students of functional analysis, applied mathematics, physics and engineering will find this an invaluable introduction to the subject. Main aim is to introduce students to the solutions of problems involving evolution equations via the theory of semigroup of operators. This course will enable the students to proceed to advanced textbooks and to many research papers devoted to the use of semigroups in the study of evolution systems.

Core Contents: Spectral Theory, Cauchy's Functional Equation, Semigroups on Banach and Hilbert spaces, Strongly continuous semigroups, Well-posedness for evolution equations, Semilinear problems.

Course Contents: Spaces and operators, spectral theory, fixed point theorem, uniformly continuous operator semigroups, semigroups on Banach spaces, semigroups on Hilbert spaces, strongly continuous semigroups, generators of semigroups, Hille-Yosida theorems, dissipative and m-dissipative operators, construction of semigroups, perturbation of generators, abstract Cauchy problems, inhomogeneous Cauchy problems, semilinear ACP, mild solutions, strong solutions.

Learning Outcomes: Students are expected to understand Spectral Theory, Cauchy's Functional Equation, Semigroups on Banach and Hilbert spaces, Strongly continuous semigroups, and applications of semigroup operator theory in differential equations and functional equations.

Text Books:

1. Bellani-Morante and A. C. McBride, Applied Nonlinear Semigroups, John Wiley & Sons (Referred as BM)
2. K-J Engel and R. Nagel, One Parameter Semigroups for Linear Evolution Equations Springer (Referred as EN)

Weekly Breakdown		
Week	***Section***	***Topics***
1	BM 1.7-1.11	Spaces and Operators, spectral Theory, Fixed Point Theorem
2	EN Chapter 1 1.1-1.4, 2.1-2.11	Cauchy's Functional Equation, Finite Dimensional Systems
3	Chapter 1 Section 3	Uniformly continuous operator semigroups, semigroups on Banach spaces, Semigroups on Hilbert spaces
4	Chapter 1 4.1 – 4.8	Multiplication Semigroups, Translation semigroups
5	Chapter 1 Section 5	Strongly continuous semigroups and its basic properties
6	Chapter 2 1.1-1.7	Construction and examples of strongly continuous semigroups
7	1.8-1.14	Generator of Semigroups and their resolvents
8	Chapter 2 2.1-2.11	Standard construction of similar semigroups, rescaled semigroups, subspace semigroups, quotient semigroups, adjoint semigroups, Product semigroups
9	**Mid Semester Exam**	
10	3.1-3.11	Hille-Yosida Generation Theorems
11	3.13-3.23	Dissipative Operators and Contractive Semigroups
13	4.1-4.15	Special classes of semigroups
14	Chapter 2 6.1-6.11 BM 2.5	Well-posedness for evolution equations, abstract Cauchy problems, Inhomogeneous abstract Cauchy problem and its strong solutions
15	Chapter 3 1.-1.15 BM 2.4	Perturbation of Generators, the Trotter-Kato theorems
16	BM 3.1-3.2	Semilinear problems
17	BM 3.3-3.4	strong solutions, mild solutions
17	Review	
18	**End Semester Exam**	

MATH-945 Lie Group Representations

Credit Hours: 3-0
Prerequisite: None

Objectives and Goals: The representation theory of Lie groups plays an important role in the mathematical analysis of the elements. In particular, the study of representations of the Lie group SO(3) leads to an explanation of the Periodic Table of the chemical elements, the study of representations of the Lie group SU(2) naturally leads to the famous Dirac equation describing the electron.
The objective of the course is to introduce the concepts of matrix Lie groups and exponentials, Lie algebras and basic representation theory. After completion of the course students are expected to be equipped with the concepts of representation theory of Lie groups and are able to apply the tools learnt in different areas like general relativity, string theory etc.

Core Contents: Matrix Lie Groups, The Matrix Exponential, Lie Algebras, Basic Representation Theory.

Course Contents: Matrix Lie Groups: Definitions, Examples, Topological Properties, Homomorphisms, Lie Groups.
The Matrix Exponential: The Exponential of a Matrix, Computing the Exponential, TheMatrix Logarithm, Further Properties of the Exponential, The Polar Decomposition.
Lie Algebras: Definitions and First Examples, Simple, Solvable, and Nilpotent Lie Algebras, The Lie Algebra of a Matrix Lie Group, Examples, Lie Group and Lie Algebra Homomorphisms, The Complexification of a Real Lie Algebra, The Exponential Map, Consequences of Theorem 3.42.
Basic Representation Theory: Representations, Examples of Representations, New Representations from Old, Complete Reducibility, Schur's Lemma, Representations of sl(2;C), Group Versus Lie Algebra Representations, A Nonmatrix Lie Group.
Course Outcomes: Students are expected to understand Matrix Lie Groups, The Matrix Exponential, Lie Algebras and Basic Representation Theory.

Learning Outcomes: Students are expected to understand Matrix Lie Groups, The Matrix Exponential, Lie Algebras, Basic Representation Theory.

Text Book: Brian C. Hall, Lie Groups, Lie Algebras, and Representations (2^{nd} Ed.), Springer International Publishing (2015).

Reference Books:

1. Andrew Baker, Matrix Groups: An Introduction to Lie Group Theory,Springer (2002).
2. MariánFecko, Differential Geometry and Lie Groups for Physicists, Cambridge University Press (2006).
3. Robert Gilmore, Lie Groups, Lie Algebras and Some of Their Applications, Dover Publications (2006).

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1,1.2	Matrix Lie Groups: Definitions, Examples.
2	1.3	Topological Properties.
3	1.4,1.5,2.1	Homomorphisms, Lie Groups. The Matrix Exponential: The Exponential of a Matrix.
4	2.2 - 2.4	Computing the Exponential, The Matrix Logarithm, Further Properties of the Exponential.
5	2.5, 3.1	The Polar Decomposition. Lie Algebras: Definitions and First Examples.
6	3.2,3.3	Simple, Solvable, and Nilpotent Lie Algebras, The Lie Algebra of a Matrix Lie Group.
7	3.4, 3.5	Examples, Lie Group and Lie Algebra Homomorphisms.
8	3.6, 3.7	The Complexification of a Real Lie Algebra, The Exponential Map.
9	**Mid Semester Exam**	
10	3.8	Consequences of Theorem 3.42.
11	4.1,4.2	Basic Representation Theory: Representations, Examples of Representations.
12	4.3	New Representations from Old.
13	4.4,4.5	Complete Reducibility, Schur's Lemma.
14	4.6	Representations of sl(2;C).
15	4.7	Group Versus Lie Algebra Representations.
16	4.8	A Nonmatrix Lie Group
17	-	Review
18	**End Semester Exam**	

MATH-946 Category Theory

Credit Hours: 3-0
Prerequisite: Fundamental knowledge of Topology & Algebra

Objectives and Goals: This course aims at introducing students to the concepts of categories, functors and natural transformations. On successful completion of this course, students will know categories, discrete objects, indiscrete objects, functors, properties of functors, natural transformations, products, co-products, equalizers, co-equalizers, pullbacks, pushouts, limits and co-limits.

Course Contents: Categories, morphisms, concrete categories, abstract categories, sections, retractions, isomorphism, monomorphisms, epimorphisms, initial objects, final objects and zero objects, functors, hom- functors, Properties of functors, natural transformations and natural isomorphisms, equalizer and coequalizer, products and coproducts, discrete and indiscrete objects, sources and sinks, pullbacks, pushouts, limit, co-limits.

Course Outcomes: Students are expected to understand:

- Categories, morphisms, abstract and concrete categories
- Sections, Retractions, Isomorphism, Mono and Epimorphism
- Initial, Final and Zero Objects
- Functors and Properties of Functors
- Natural transformations and Natural isomorphism
- Equalizer, Coequalizer, Product and Coproduct
- Discrete and Indiscrete objects
- Sources and Sinks
- Pullbacks and Pushouts
- Limits and Colimits

Text Books:
S. Awodey, "Category Theory", Oxford University Press (2nd edition), 2010.
J. Adamek, H. Herrlich, and G. E. Strecker, "Abstract and Concrete Categories, The Joy of Cats", Dover Publications, 2009.

Reference Books:
1. G. Preuss, "Foundations of Topology", Kluwer Academik Publisher, 2002.
2. S. Mac Lane, "Categories for working mathematicians", Springer, 2nd Edition, 1997.
3. D. I. Spivak, "Category theory for the Sciences", MIT press, 2013
4. M. Barr and C. Wells, "Category theory for Computing Science", Prentice hall international UK, 1990

Weekly Breakdown	
Week	***Topics***
1	Sets, Classes and conglomerates categories, Morphisms
2	Concrete Categories, Abstract Categories
3	Section, Retractions, Monomorphisms
4	Epimorphisms and Isomorphisms
5	Functors, Hom-functors
6	Properties of functors
7	Initial objects, Final objects and Zero objects
8	Fixed morphisms, Zero morphisms and Point categories
9	**Mid Semester Exam**
10	Natural transformation, Natural isomorphisms
11	Discrete and Indiscrete objects
12	Equalizer, Coequalizer
13	Products and Coproducts
14	Pullbacks, Pushouts
15	Sources and Sinks
16	Limit, Co-limits,
17	Review
18	**End Semester Exam**

MATH-949 Combinatorics

Credit Hours: 3-0
Prerequisites: None

Course Objectives: This course is for the students in MS/ PhD Mathematics program. The main objective of this course is to understand countable discrete structures. The main educational objectives of this course are:
To introduce the discrete structures and discrete mathematical models
To model, analyze, and to solve combinatorial and discrete mathematical problems.
It is also aimed to develop the ability in students to apply these techniques for solving the practical problems in optimization, computer science and engineering as well as to apply combinatorial techniques in other disciplines of mathematics like algebra, graph theory and geometry etc.

Core Contents: Classical Techniques, Generating functions, Recurrence relation, Combinatorial Numbers, Partition of Integers, Inclusion-Exclusion Principal and applications, Polya's enumeration theory, Chromatic Polynomials of graphs

Detailed Course Contents: Classical Techniques: Two Basic counting principals, Binomial, Multinomial numbers and multinomial formula, combinations with or without repetitions, Permutations and permutation with forbidden positions; Brief Introduction to graphs/discrete structures. Generating Functions: Generating Function Models, Calculating Coefficients of Generating Functions, Exponential Generation Functions. Partition of Integers: Partitions of integers (their properties, recurrence relations, generating functions). Recurrence Relation: Recurrence Relation Models, Divide-and-Conquer Relations, Solution of Linear Recurrence Relations, Solution of Inhomogeneous Recurrence Relations, Solution with Generating Functions. Inclusion-Exclusion Principals: Counting with Venn diagrams, Inclusion, Inclusionformula and its forms, Applications of Inclusion-Exclusion. Combinatorial Numbers: Stirling, Bell, Fibonacci and Catalan numbers (their recurrence relations, generating functions and applications to enumeration problems in graph theory and geometry). Polya enumeration theory: Equivalence and symmetry groups, Burnside's Theorem. Chromatic Polynomials: Fundamental Reduction Theorem, Chromatic Equivalence, Chromatic Uniqueness

Course Outcomes: This course is specially designed for students who want to choose pure mathematics as their specialty in general and more specifically who want to opt discrete mathematics as their research area. On successful completion of this course, students will be able

- To understand the fundamental structures and techniques of combinatorial mathematics and importance of combinatorial techniques in comparison with other techniques
- To explore the logical structure of mathematical problems,
- To develop problem solving skills in combinatorial related problems and their applications.

Text Book: Alan Tucker, Applied Combinatorics (4th Edition, 2002) John Wiley and Sons.

Reference Books:

1. John M. Harris, Jeffry L. Hirst, Micheal J. Mossinghoff, Combinatorics and Graph Theory, Springer, 2nd Edition, 2008.
2. V. Krishnamurthy, Combinatorics, theory and applications, Ellis Horwood Publ., Chichester,1986.
3. R . A. Brualdi, Introductory Combinatorics (5th Edition), 2010, Prentice Hall.

Weekly Breakdown		
Week	*Section*	*Topics*
1	5.1, 5.2, 5.5	Two Basic Counting Principles, Simple Arrangements and Selections, Binomial Coefficients and Binomial formula, Multinomial formula
2	5.3, 5.4, 5.5	Arrangements and Selections with Repetitions, Multinomial Coefficients and multinomial formula, The Pigeonhole Principle, Distributions, Binomials Identities
3	6.1, 6.2, 6.3	Generating Functions Models, Calculating Coefficients of Generating functions and applications, Exponential Generating Functions
4	7.1, 7.2	Recurrence Relation Models, Divide-and-Conquer relations
5	7.3,7.4	Solutions of Linear Recurrence Relations, Solution of Inhomogeneous RecurrenceRelations
6	7.5	Solutions with Generating Functions
7	8.2	Counting with Venn Diagrams, Inclusion-Exclusion principle and applications
8	8.3	Permutations with forbidden positions
9	**Mid Semester Exam**	
10	2.6.4, 2.6.6	Stirling numbers (First kind and second kind) and Bell numbers (their recurrence relations, generating functions), applications of these numbers to enumeration problems in graph theory and geometry
11	2.8.2, 2.8.3	Fibonacci and Catalan numbers (recurrence relations,
12	9.1, 9.2	generating functions) and applications
13	9.3	Equivalence and symmetry groups, Burnside's Theorem,
14	6.3	Partitions of integers (their properties, recurrence relations, generating functions)
15	9.4	Polya's Theorem and applications
16	J. M. Harris 1.6.4	Chromatic polynomials in graph colorings (properties and the fundamentalreduction theorem), Chromatic Equivalence and chromatic Uniqueness
17	-	Review
18	**End semester Exam**	

MATH-955 General Relativity and Cosmology

Credit Hours: 3-0
Prerequisite: None

Course Objectives: General Relativity (GR) is a physical theory of gravitation invented by Albert Einstein in the early twentieth century. The theory has strong mathematical setup, has immense predictive power, and has successfully qualified several experimental/observational experiments of astrophysics and cosmology. Black holes and relativistic cosmology are two main applications of GR. It is intended that GR and its major applications and achievements be discussed in the manner they deserve.

Core Contents: Special relativity revisited, Electromagnetism, The gravitational field equations, The Schwarzschild geometry, Schwarzschild black holes, Kerr metric, Further spherically symmetric geometries.

Detailed Course Contents: Special relativity revisited: Minkowskispacetime in Cartesian coordinates, Lorentz transformations, Cartesian basis vectors, Four-vectors and the lightcone, Four-vectors and Lorentz transformations, Four-velocity, Four-momentum of a massive particle, Four-momentum of a photon, The Doppler effect and relativistic aberration, Relativistic mechanics, Free particles, Relativistic collisions and Compton scattering, Accelerating observers, Minkowskispacetime in arbitrary coordinates.
Electromagnetism: The electromagnetic force on a moving charge, The 4-current density, The electromagnetic field equations, Electromagnetism in the Lorenz gauge, Electric and magnetic fields in inertial frames, Electromagnetism in arbitrary coordinates, Equation of motion for a charged particle, Electromagnetism in a curved spacetime.
The gravitational field equations: The energy–momentum tensor, The energy–momentum tensor of a perfect fluid, Conservation of energy and momentum for a perfect fluid, The Einstein equations, The Einstein equations in empty space, The weak-field limit of theEinstein equations, The cosmological-constant term.
The Schwarzschild geometry: General static isotropic metric, Schwarzschild solution, Birkhoff's theorem, Gravitational redshift, geodesics in Schwarzschild geometry, radial trajectories of massive particles, Circular motion of massive particles, stability of massive particle orbits, trajectories of photons, circular motion of photons, stability of photon orbits, Experimental tests of general relativity: Precession of planetary orbits, The bending of light, Accretion discs around compact objects.
Schwarzschild black holes: singularities in Schwarzschild metric, radial photon worldlines, radial particle worldliness in Schwarzschild coordinates, Eddington Finkelstein coordinates, black hole formation, Spherically symmetric collapse of dust, tidal forces near a black hole, Kruskal coordinates, wormholes and Einstein Rosen bridge, The Hawking effect of blackhole evaporation.
Further spherically symmetric geometries: Spherically symmetric geometries: metric for stellar interior, relativistic equations of stellar structure, Schwarzschild interior solution, metric outside a spherically symmetric charged mass, Riessner-Nordstrom geometry and solution, Radial photon trajectories in RN geometry, radial massive particle trajectories.
Kerr metric: The Kerr metric, Limits of the Kerr metric, Ker Neumann Metric (handouts). The Friedmann–Robertson–Walker geometry: The cosmological principle, synchronous commoving coordinates, homogeneity and isotropy of the universe, maximally symmetric 3-space, Friedmann-Robertson-Walker metric, geometrical properties of FRW metric, The cosmological redshift, The Hubble and deceleration parameters, Components of the cosmological fluid, Cosmological parameters, The cosmological field equations, General

dynamical behaviour of the universe, Evolution of the scale factor, Analytical cosmologicalmodels.

Learning Outcomes: Students will understand of the theory and predictions of Einstein's general relativity. Students will be capable to read research papers and initiate research in general relativity. Students will be able to understand the dynamical evolution of the universeby studying cosmology.

Text Book: M.P. Hobson, G.P. Efstathiou, A.N. Lasenby, General Relativity, Cambridge University Press (2007).

Weekly Breakdown		
Week	***Section***	***Topics***
1	5.1-5.7	Special relativity revisited: Minkowski space time in Cartesian coordinates, Lorentz transformations, Cartesian basis vectors, Four-vectors and the lightcone, Four-vectors and Lorentz transformations, Four-velocity, Four-momentum of a massive particle.
2	5.8-5.14	Four-momentum of a photon, The Doppler effect and relativistic aberration, Relativistic mechanics, Free particles, Relativistic collisions and Compton scattering, Accelerating observers, Minkowski space time in arbitrary coordinates.
3	6.1-6.4	Electromagnetism: The electromagnetic force on a moving charge, The 4-current density, The electromagnetic field equations, Electromagnetism in the Lorenz gauge.
4	6.5-6.7	Electric and magnetic fields in inertial frames, Electromagnetism in arbitrary coordinates, Equation of motion for a charged particle, Electromagnetism in a curved spacetime.
5	8.1-8.7	The gravitational field equations: The energy–momentum tensor, The energy–momentum tensor of a perfect fluid, Conservation of energy and momentum for a perfect fluid, The Einstein equations, The Einstein equations in empty space, The weak-field limit of the Einstein equations, The cosmological-constant term.
6	9.1-9.7	The Schwarzschild geometry: General static isotropic metric, Schwarzschild solution, Birkhoff's theorem, Gravitational redshift, geodesics in Schwarzschild geometry, radial trajectories of massive particles.
7	9.8-9.13	Circular motion of massive particles, stability of massive particle orbits, trajectories of photons, circular motion of photons, stability of photon orbits.
8	10.1, 10.2, 10.4	Experimental tests of general relativity: Precession of planetary orbits, The bending of light, Accretion discs around compact objects.
9	**Mid Semester Exam**	
10	11.1 - 11.6	Schwarzschild black holes: singularities in Schwarzschild metric, radial photon worldlines, radial particle worldliness in Schwarzschild coordinates, Eddington Finkelstein coordinates, black hole formation.
11	11.7 - 11.11	Spherically symmetric collapse of dust, tidal forces near a black hole, Kruskal coordinates, wormholes and Einstein Rosen bridge, The Hawking effect of black hole evaporation.
12	12.1-12.6	Further spherically symmetric geometries: Spherically symmetric geometries: metric for stellar interior, relativistic equations of stellar structure, Schwarzschild interior solution, metric outside a spherically symmetric charged mass, Riessner-Nordstrom geometry and solution
13	12.7-12.8 13.5, 13.6	Radial photon trajectories in RN geometry, radial massive particle trajectories, Kerr metric: The Kerr metric, Limits of the Kerr metric, Ker Neumann Metric (handouts).
14	14.1-14.7	The Friedmann–Robertson–Walker geometry: The cosmological principle, synchronous commoving coordinates, homogeneity and isotropy of the universe, maximally symmetric 3-space, Friedmann-Robertson-Walker metric, geometrical properties of FRW metric.
15	14.9, 14.10	The cosmological redshift, The Hubble and deceleration parameters.
16	15.1-15.6	Components of the cosmological fluid, Cosmological parameters, The cosmological field equations, General dynamical behaviour of the universe, Evolution of the scale factor, Analytical cosmological models.
17	-	Review
18	**End Semester Exam**	

MATH-956 Finite Volume Method

Credit Hours: 3-0
Prerequisite: None

Course Objectives: This course aims on a powerful class of numerical methods for approximating solution of hyperbolic partial differential equations, including both linear problems and nonlinear conservation laws.

Core Contents: Conservation laws, Finite volume methods, Multidimensional problems. Linear waves and discontinuous media. The advection equation. Diffusion and the advection–diffusion equation, Nonlinear equations in fluid dynamics. Linear acoustics, Sound waves. Hyperbolicity of linear systems, Variable-coefficient hyperbolic systems. Solution to the Cauchy problem. Superposition of waves and characteristic variables, Left eigenvectors, Simple waves, Acoustics, Domain of dependence and range of influence. Discontinuoussolutions, The Riemann problem for a linear system. Coupled acoustics and advection,Initial–boundary-value problems.
General formulation for conservation laws, A numerical flux for the diffusion equation, Necessary components for convergence, The CFL condition. An unstable flux, The Lax– Friedrichs method, The Richtmyer two-step Lax–Wendroff method, Upwind methods, The upwind method for advection. Godunov's method for linear systems, The numerical flux function for Godunov's method. Flux-difference vs. flux-vector splitting, Roe's method. The Lax–Wendroff method, The beam–warming method, Preview of limiters. Choice of slopes, Oscillations, Total variation. Slope-limiter methods, Flux formulation with piecewise linear reconstruction, Flux limiters, TVD limiters

Course Outcomes: Students are expected to understand the various variants of the of finite volume method and its applications to problems like:

- Linear waves and discontinuous media.
- Diffusion and the advection–diffusion equation.
- Coupled acoustics and advection.

Text Book: Randall J. Leveque, Finite Volume Methods for Hyperbolic, Problems, Cambridge University Press, (2004)

Reference Books: F. Moukalled, L. Mangani, M. Darwish, "The Finite Volume Method in Computational Fluid Dynamics'', Springer, 2016

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1-1.3	Conservation laws, Finite volume methods, Multidimensional problems.
2	1.4.2.1	Linear waves and discontinuous media. The advection equation.
3	2.2,2.6	Diffusion and the advection–diffusion equation, Nonlinear equations in fluid dynamics.
4	2.7,2.8	Linear acoustics, Sound waves.
5	2.9,2.10,3.1	Hyperbolicity of linear systems, Variable-coefficient hyperbolic systems. Solution to the Cauchy problem.
6	3.2-3.6	Superposition of waves and characteristic variables, Left eigenvectors, Simple waves, Acoustics, Domain of dependence and range of influence.
7	3.7,3.8	Discontinuous solutions, The Riemann problemfor a linear system
8	3.10,3.11	Coupled acoustics and advection, Initial–boundary-value problems.
9	**Mid Semester Exam**	
10	4.1-4.4	General formulation for conservation laws, A numerical flux for the diffusion equation, Necessary components for convergence, The CFL condition.
11	4.5-4.9	An unstable flux, The Lax–Friedrichs method, The Richtmyer two-step Lax–Wendroff method, Upwind methods, The upwind method for advection.
12	4.10,4.11	Godunov's method for linear systems, The numerical flux function for Godunov's method.
13	4.13, 4.14	Flux-difference vs. flux-vector splitting, Roe's method
14	6.1-6.3	The Lax–Wendroff method, The beam–warming method, Preview of limiters.
15	6.5-6.7	Choice of slopes, Oscillations, Total variation.
16	6.9-6.12	Slope-limiter methods, Flux formulation with piecewise linear reconstruction, Flux limiters, TVD limiters
17	-	Review
18	**End Semester Exam**	

MATH-957 Algebraic Topology

Credit Hours: 3-0
Prerequisite: None

Course Objectives: The objective of this course is to introduce the basic concepts about homotopy and homotopy type, fundamental group and covering spaces to use in his/her research and in other areas like differential geometry, algebraic geometry, physics etc.

Core Contents: Connected spaces, Path connected spaces, Compact spaces, Homotopy equivalence, Path homotopy, Fundamental group, Induced homomorphism, Van Kampen's Theorem, Covering spaces, Singular homology, Homotopy invariance, Homology long exact sequence.

Detailed Course Contents: Topological spaces, Closure and interior points, Bases, Continuity, Homeomorphism, Compactness, Path connectedness, Connectedness, Relationship between connectedness and path connectedness, History of algebraic topology, Homotopy, Homotopy classes, Path homotopy, Fundamental group, Fundamental group of a circle, Induced homomorphism, Van Kampen's theorem, Covering space, Universal cover, Classification of Covering spaces, Deck transformation, Covering space action, Idea of Homology, Simplicial homology, Singular homology, Chain homotopy, Homotopy invariance of Homology, Exact sequence, Degree and Cellular homology, Application of homology in group.

Learning Outcomes: On successful completion of this course students will be able to:

- Understand the definitions of homotopy, homotopy equivalence, fundamental group.
- Understand methods to construct and classify covering spaces for known spaces, and for other spaces whenever it is possible.
- Understand the relation between singular homology and fundamental group.
- Understand the homology of a group.

Textbooks:
Andrew H. Wallace, (AW) "An Introduction to Algebraic Topology", Dover Publisher, (2007)
Allen Hatcher, (AH) "Algebraic Topology", Cambridge University Press, (2002)

Reference Books:
1. Joseph J. Rotman, "An Introduction to Algebraic Topology", Springer, (1988)
2. J. Peter May, "A Concise Course in Algebraic Topology", Chicago University Press, (1999)
3. R. Brown, "Topology and Groupoids", Book Surge Publishing, (2006)

Weekly Breakdown		
Week	***Section***	***Topics***
1	(AW) 2.1-2.8	Definition of Topology, Open sets, Subspace, Limit and Closure points, Bases
2	3.1-3.2	Continuous Mapping, Homeomorphism, Compactness
3	3.3	Pathwise Connectedness and Related Results
4	3.4	Connectedness, Examples, Relationship between Connectedness and Pathwise connectedness
5	4.1	History of Algebraic Topology, Homotopy and Results, Homotopy
6	4.2	Homotopy classes, Path Homotopy and Results
7	4.3-4.4	Fundamental Groups, Fundamental group of a Circle
8	(AH) 1.1.3	Induced Homomorphism and Results
9	**Mid Semester Exam**	
10	1.2.1, 1.2.2	Free Product of Groups, Van Kampen's theorem and Application
11	1.3.1, 1.3.2	Covering Spaces and Lifting Criterion, Universal Cover
12	1.3.3	Classification of Covering space, Deck Transformation and Group actions
13	2.1.1, 2.1.2	Homology, Types of Homology, Simplicial Homology
14	2.1.3	Singular Homology, Chain Homotopy
15	2.1.4-2.1.5	Homotopy invariance of Homology, Exact Sequence
16	2.2.1-2.2.2,	Degree and Cellular homology, Homology of a group
17	-	Review
18	**End Semester Exam**	

MATH-XXX Finite Difference Methods for Differential Equations

Credit Hours: 3-0
Prerequisite: None

Course Objectives: The objective of this course is to find numerical solution of ordinary and partial differential equations by finite difference method. The basics and advanced topics relevant to finite difference method will be covered. These topics will be very useful for the students who opts for the research topic in differential equations. Not only students will be given theoretical aspects of numerical schemes but also programming experience in MATLAB will be helpful.

Core contents: Finite difference approximations, boundary value problems, elliptic equations, iterative method for sparse system, advection equations and hyperbolic systems

Course Contents: Truncation errors, finite difference approximations, the heat equation, the steady-state problem, local truncation error, global error, stability, consistency, steady-state heat conduction, Jacobi and Gauss-Seidal, rate of convergence, The Arnoldi process and GMRES algorithm, Advection equation, Leapfrog method, Lax-Friedrichs, The Lax-Wendroff method, Upwind methods, Von Neumann analysis, The Courant-Friedrichs-Lewy condition

Course Outcomes: After studying this subject the students will be able to:

- Compute numerical solution of ODEs and PDEs by finite difference method
- Solve sparse linear system by iterative schemes
- Program numerical solutions in MATLAB

Textbook: Finite Difference Methods for Ordinary and Partial Differential Equations by Randall J. Le Veque, Publisher: Siam, 2007.

Reference Books

1. Applied Numerical Analysis by Curtis F. Gerald and Patrick O. Wheatley, 7th Edition, Publisher: Pearson, 2003.
2. Numerical Methods for Engineers by Steven C Chapra and Raymond P Canale, 6th Edition, Publisher: McGraw-Hill, 2009.
3. Finite Difference Computing with PDEs: A Modern Software Approach by Hans Petter Langtangen and Svein Linge, Ist Edition, Publisher: Springer, 2017.

Weekly Breakdown		
Week	*Sections*	*Topic*
1	1.1, 1.2, 1.3	Truncation errors, Deriving finite difference approximations, Second order derivatives
2	2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 2.10	The heat equation, Boundary conditions, The steady-state problem, A simple finite difference method, Local truncation error, Global error, Stability, Consistency, Convergence, Stability in the 2- norm

3	2.15, 2.16, 2.16.1, 2.16.2, 2.16.3,	A general linear second order equation, Nonlinear equations, Discretization of the nonlinear boundary value problem, Nonuniqueness, Accuracy on nonlinear equations
4	3.1, 3.2, 3.3	Steady-state heat conduction, The 5-point stencil for the Laplacian, Ordering the unknowns and equations
5	3.4, 3.5, 3.6, 3.7, 3.7.1	Accuracy and stability, The 9-point Laplacian, Other elliptic equations, Solving the linear system, Spare storage in MATLAB
6	4.1, 4.2, 4.2.1, 4.2.2	Jacobi and Gauss-Seidal, Analysis of matrix slitting methods, Rate of convergence, Successive overrelaxation
7	4.4 4.4.1 4.4.2	The Arnoldi process and GMRES algorithm, Krylov methods based on three term recurrences, Other applications of Arnoldi
8	4.5	Newton-Krylov methods for nonlinear problems
9	**Mid Semester Exam**	
10	4.6	Multigrid methods
11	4.6.1, 4.6.2	Slow convergence of Jacobi, The multigrid approach
12	10.1 10.2 10.2.1	Advection, Method of lines discretization, Forward Euler time discretization,
13	10.2.2 10.2.3	Leapfrog, Lax-Friedrichs
14	10.3 10.3.1 10.4 10.4.1 10.4.2	The Lax-Wendroff method, Stability Analysis, Upwind methods, Stability Analysis, The Beam-Warming method
15	10.5 10.6 10.7	Von Neumann analysis, Characteristic tracing and interpolation, The Courant-Friedrichs-Lewy condition
16	10.8 10.9 10.10	Some numerical results, Modified equations, Hyperbolic systems
17	-	Review
18	**End Semester Exam**	

STAT-806 Statistical Learning

Credit Hours: 3-0
Prerequisite: None

Aims and Objectives: Statistics is supporting tool, which can assist MS/PhD Mathematics students working on diverse data structures. Moreover, statistical learning can help MS/PhD Mathematics students in modeling the real life applications. This course will help to understand and model the statistical data of diverse structures. This course aims at introducing students to the concepts of statistical learning with focus on regression, classification and clustering. On successful completion of this course, students will know statistical learning, regression, classification and clustering.

Course Contents: Modern data analysis, methods where fewer assumptions (such as a linear relation between response and explanatory variables) are made and where instead data determine the relation. Nearest neighbor methods, kernel smoothing and generalized additive models. Statistical classification, classical classification methods, advanced methods based on modern regression methods. Structures in data, Data mining i.e. Learning from data

Textbook: Hastie T., Tibshirani R., and Friedman, J., Introduction to Statistical Learning, Springer (2013).

Reference books:

1. James, W., Hastie, T. and Tibshirani, R. An Introduction to Statistical Learning, with Applications in R, Springer (2003)
2. Trevor, H., Robert T. and Martin W. Statistical Learning with Sparsity: The Lasso and Generalizations, CRC Press (2015).
3. Hastier, T.J. and Tibshirani R.J. Generalized Additive Models 2nd Edition.

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Weekly Breakdown		
Week	***Section***	***Topics***
1	1 (2.1-2.2)	Supervised learning and variable types
2	2 (3.2)	Linear Regression
3	3 (3.3)	Model/Subset Selection
4	4 (3.4)	Shrinkage Methods
5	5 (3.5)	Principal Components and Partial Least Squares Regression
6	7 (4.2)	Classification with Linear Regression of an Indicator Matrix
7	8 (4.3)	Linear Discriminant Analysis
8	9 (4.4)	Logistic Regression
9	**Mid Semester Exam**	
10	10 (4.5, 12.2)	Separating Hyperplanes/The Support Vector Classifier .
11	11 (12.4-12.6)	Various Discriminant Analysis
12	13 (7.2-7.4)	Bias, Variance, Error
13	14 (7.10-7.11)	Cross validation and Bootstrap
14-15	15 (14.3)-16 (14.3)	Cluster Analysis
16-17	-	Revision
18	**End Semester Exam**	

Annex B

PhD. Mathematics Program

Program Description

1. The aim of PhD program in Mathematics is to impart quality education and inculcate research abilities so that the graduates are ready to be part of the much-needed quality human resource in the field of mathematics. In order to provide the potential PhD students with a broad base that will enable them to conduct worthwhile research in some mainstream area, the school offers a wide range of courses from its major thrust areas, which include Differential Equations, Mathematical Physics, Discrete Mathematics, Fluid Mathematics, Computational Mathematics, Algebra, Topology and Functional Analysis.

2. The PhD program consists of 18 credit hours coursework and 30 credit hours thesis. There are also seminar courses that must be cleared but carry no credits. After completing course work, PhD students are required to clear a comprehensive exam as per the NUST policy that includes defence of synopsis which forms the basis of PhD research work.

Rationale for Revision

4. As per NUST Policy, all programs are required to be revised after the completion of every 4 years. The existing Ph.D. curriculum was revised in 2019. Therefore, the current revision is initiated keeping in view the national and international practices. In this revision, the contents of some courses are revised. Also, some new courses are included.

Eligibility Criteria

5. In addition to NUST laid down criteria, MS/MPhil/Equivalent in the following

- Mathematics
- Physics
- Computer Science
- Computational Science and Engineering
- Electronics
- Statistics with BS in Mathematics
- Electrical Engineering
- Mechanical Engineering
- Mechatronics Engineering
- Computer Engineering
- Software Engineering
- Aerospace Engineering
- Avionics Engineering

Reading and Research Courses

4. Reading and Research courses are approved courses for Ph.D. programs at NUST. Specialized reading courses supplement regular graduate course offerings, allowing Ph.D. students to have deeper knowledge in a particular research area. A supervisor decides to offer these courses for his/her Ph.D. student if the student desires to explore a subject area that is not offered in a regular semester or if the student wishes to have an adequate understanding of some topics related to his/her area of research.

(a). **Course Contents**: The supervisor interested in offering a Reading and Research course to his/her Ph.D. student will prepare a course outline that includes course objectives, learning outcomes, course contents, and week-wise course contents. In addition, relevant text/reference books or research papers will be mentioned in the course outline.

(b). **Approval:** The supervisor will submit the course contents in a standard format containing a detailed week-wise breakdown to the concerned HoD for approval usually one week before the commencement of the regular semester. After approval from HoD, the instructor/supervisor will submit a copy of the course contents to the Program Coordinator. The Program Coordinator will register the course on the Qalam and add it to the timetable.

(c). **Timetable:** Reading and Research Courses will be included in the timetable and taught in the classrooms.

(d). **Examinations and Grading**: Evaluation and assessment of students shall be conducted as per NUST examination rules for PG courses, that is, regular conduct of quizzes, and assignments apart from scheduled Mid- Semester and End semester Exams. Students will be graded as per NUST approved procedure in vogue for PG courses.

(e). **Credit Hours:** For a student, a Reading and Research course will count 3 credit hours and for the instructor, it will count as 1 credit hour per course.

Input from industry and Academia

5. Input on the revised curriculum has been sought from the following academia and industry representatives in an advisory board meeting held on October 24, 2022.

S.No	Name	Designation/ Institution
1	Prof. Dr. Muhammad Sajid	Professor of Department of Mathematics IIU Islamabad
2	Prof. Dr. Shahid Hamid	Professor/ Dean of Natural Sciences, QAU Islamabad.
3	Mr. Tariq Mehmood Khan	CEO Redox (SMC PVT) LTD Islamabad

Minutes of the advisory board meeting are attached.
Suggestions/inputs from the following alumnae have been incorporated in the working paper.
(i) Hafiz Muhammad Fahad
(ii) Zain ul Abdeen

Timeframe of commencement

6. The revised PhD Mathematics program will be implemented for Fall 2023 and onward batches.

Approved by DBS/FBS

7. The working paper for the revision of the PhD Mathematics program was discussed in several DCRC meetings held from time to time, in the DBS held on October 27th, 2022, and in the FBS held on November 4th, 2022. The FBS supported the revised program and recommended for further deliberation in the UCRC meeting and ACM. Minutes of the FBS are enclosed.

Approved by DBS/FBS

7. The working paper for the revision of the PhD Mathematics program was discussed in several DCRC meetings held from time to time, in the DBS held on October 27th, 2022, and in the FBS held on November 4th, 2022. The FBS supported the revised program and recommended for further deliberation in the UCRC meeting and ACM. Minutes of the FBS are enclosed.

Prepared by

Prof. Matloob Anwar (Head of DCRC) ________ Prof. Mujeeb ur Rehman ________

Dr. Muhammad Ishaq ________ Dr. Muhammad Qasim ________

Checked by

Prof. Meraj Mustafa Hashmi (HoD Mathematics) ________

Prof. Rashid Farooq (Principal SNS)

Changes in PhD-Mathematics Courses

S. No.	Type of change	No. of courses
1	Courses revised	02
2	Courses with no change	42
3	New courses included	03
4	Courses Discarded (Replaced with new courses)	03

Details of changes

S. No	Code	Subject	CHs	Details of Changes			
				Code change	Title change	Contents revised	Remarks
Revised courses							
1	MATH-801	Algebra	3-0	No	No	Yes	Contents revised
2	MATH-807	Commutative Algebra	3-0	No	No	Yes	Contents revised
Courses with no change							
1	MATH-803	Geometry	3-0	-	-	-	
2	MATH-817	Advanced Functional Analysis	3-0	-	-	-	
3	MATH-818	Theory of Ordinary DifferentialEquations	3-0	-	-	-	
4	MATH-819	Analysis of Fractional Differential Equations	3-0	-	-	-	
5	MATH-820	Calculus of Variations andOptimal Control	3-0	-	-	-	
6	MATH-821	Analytical Approximate Solutions of ODEs	3-0	-	-	-	
7	MATH-822	Mathematical Modelling-I	3-0	-	-	-	
8	MATH-903	Partial Differential Equation-I	3-0	-	-	-	
9	MATH-904	Partial Differential Equation-II	3-0	-	-	-	
10	MATH-905	Symmetry Methods for Differential Equation-I	3-0	-	-	-	
11	MATH-906	Symmetry Methods for Differential Equation-II	3-0	-	-	-	
12	MATH-908	Fixed Point Theory	3-0	-	-	-	
13	MATH-909	Continuum Mechanics-I	3-0	-	-	-	
14	MATH-910	Continuum Mechanics-II	3-0	-	-	-	
15	MATH-911	Special Function	3-0	-	-	-	
16	MATH-941	Graph Theory	3-0	-	-	-	
17	MATH-943	Convex Analysis	3-0	-	-	-	
18	MATH-944	Semigroup Theory of Operators	3-0	-	-	-	
19	MATH-945	Lie Group Representations	3-0	-	-	-	
20	MATH-946	Category Theory	3-0	-	-	-	
21	MATH-949	Combinatorics	3-0	-	-	-	

22	MATH-951	Mathematical Modelling-II	3-0	-	-	-	
23	MATH-955	General Relativity and Cosmology	3-0	-	-	-	
24	MATH-956	Finite Volume Method	3-0	-	-	-	
25	MATH-957	Algebraic Topology	3-0	-	-	-	
26	ME-881	Advanced Fluid Mechanics	3-0	-	-	-	
27	PHY-801	Classical Mechanics	3-0	-	-	-	-
28	PHY-803	Quantum Mechanics	3-0	-	-	-	-
29	PHY-805	Electromagnetism	3-0	-	-	-	-
30	PHY-806	Thermal Physics	3-0	-	-	-	-
31	PHY-902	Quantum Field Theory-I	3-0	-	-	-	-
32	PHY-907	General Relativity	3-0	-	-	-	-
33	PHY-908	Cosmology-I	3-0	-	-	-	-
34	PHY-912	Relativistic Astrophysics	3-0	-	-	-	-
35	PHY-914	Particle Physics-I	3-0	-	-	-	-
36	PHY-920	Classical Field Theory	3-0	-	-	-	-
37	MATH-960	Reading and Research-I	3-0	-	-	-	
38	MATH-961	Reading and Research-II	3-0	-	-	-	
39	MATH-982	Seminar Delivered-G	0	-	-	-	
40	MATH-984	Seminar Delivered-R	0	-	-	-	
41	SEM/WKS P-997	Seminar/Workshop	1-0	-	-	-	
42	MATH-999	PhD Thesis	30	-	-	-	
New courses included							
1	MATH-XXX	Computational Mathematics	3-0	-	-	-	Replacement of MATH-804 Differential Equations
2	MATH-XXX	Advanced Topology	3-0	-	-	-	-
3	MATH-XXX	Finite Difference Methods for Differential Equations	3-0	-	-	-	-
Courses discarded							
1	MATH-804	Differential Equations	3-0	-	-	-	Replaced with MATH-XXX Computational Mathematics
2	MATH-802	Analysis	3-0	-	-	-	Replaced with already approved elective course MATH-817 Advanced Functional Analysis
3	MATH-XXX	Numerical Methods for Differential Equations	3-0	-	-	-	Replaced with MATH-XXX Finite Difference Methods for Differential Equations

Detailed Course Contents – Ph.D. Mathematics

MATH-801 Algebra

Credit Hours: 3-0
Prerequisite: None

Objectives and Goals: This course will provide a base for many subjects in modern Algebra such as commutative algebra, algebraic geometry, algebraic number theory, computational commutative algebra, multiplicative ideal theory, homological algebra and combinatorial commutative algebra and representation theory.

Core Contents: Groups, group actions and permutation representations, class equation of group, Sylow's theorems, simple groups, direct product and finitely generated abelian groups, rings, ideals, Euclidean domains, principal ideal domains, unique factorization domains.

Detailed Course Contents: Groups, dihedral groups, symmetric groups, matrix groups, the Quaternion group, homomorphism and isomorphism, subgroups generated by a subset of a group, the lattice of subgroups of a group, Fibers of a group homomorphism, quotient groups, group actions, group actions and permutation representations, group acting on themselves by left multiplication, group acting on themselves by conjugation, the class equation, the Sylow's theorems, simple groups, conjugacy in symmetric groups, the fundamental theorem of finitely generated abelian groups, rings, subrings, ideals, polynomial rings, quotient rings, ring homomorphism, properties of ideals, integral domains, prime and irreducible elements, Euclidean domains, principal ideal domains, unique factorization domains, polynomial rings over fields, polynomial rings that are unique factorizations

Course Outcomes: On successful completion of this course, students will know groups, subgroups, group action, factor groups, number of all possible abelian groups up to isomorphism for a given order, Sylow theorems, application to simplicity of groups, rings, subrings, ideals, polynomial rings, reducible and irreducible polynomials over certain rings, integral domains, Euclidean domains, principal ideal domains, unique factorization domains

Textbook: David S. Dummit, Richard M. Foote, Abstract Algebra, 3rd Ed., John Wiley &Sons.

Reference books:

1. N. Herstein, Topics in Algebra, John Wiley and Sons.
2. W. Keith Nicholson, Introduction to Abstract Algebra, (3rd edition), 2007, John Wiley & son

Weekly Breakdown

Week	*Section*	*Topics*
1	Sec. 1.2-1.6	Group of symmetries of a geometric object, examples, presentation of a group, matrix groups, the quaternion group, group homomorphisms
2	Sec. 1.7	Group actions, examples of group actions, permutation representation associated to the given action. faithful and transitive actions.
3	Sec. 2.1, 2.2, 2.3	Subgroups, centralizers, and normalizers. Stabilizers and kernels of the group actions. Cyclic groups and cyclic subgroups.
4	Sec. 2.4, 2.5	Subgroups generated by a subgroup of a group, the lattice of the subgroups of a group.
5	Sec. 3.1	Fibers of a group homomorphism and related theorems, quotient group using fibers of a group homomorphism, quotient group by a normal subgroup.
6	Sec. 3.2	Lagrange theorem and its converse, Cauchy's theorem, composition of two subgroups and related results.
7	Sec. 3.3, 4.1	Isomorphism theorems, the correspondence theorem and its applications to factor group. Group action and permutation representations.
8	Sec. 4.2	Orbit stabilizer theorem, group acting on themselves by left multiplication, Smallest prime index theorem.
9	**Mid Semester Exam**	
10	Sec. 4.3	Group acting on themselves by conjugation, the class equation of a group and applications, conjugacy in $S_{n.}$
11	Sec. 4.5	Proofs of Sylow's theorems using group action, applications of Sylow's theorems to simple groups.
12	Sec. 5.1, 5.2	Direct products, the fundamental theorem for finitely generated abelian groups.
13	Sec. 7.1, 7.2	Rings, matrix ring, group ring, the ring of residue classes modulo *n*, polynomial ring in several variables, integral domains, fields.
14	Sec. 7.3	Ideals, quotient rings, ring homomorphism, isomorphism theorems for rings, the correspondence theorem for rings and applications to quotient rings.
15	Sec. 7.4, 8.1, 8.2	Properties of ideals, characterization of prime and maximal ideals. Norms on integral domains, division algorithms for integral domains, examples, principal ideal domains, examples.
16	Sec. 8.3	Prime and irreducible elements, examples, unique factorization domains, examples.
17		Review
18	**End Semester Exam**	

MATH-803 Geometry

Credit Hours: 3-0
Prerequisite: None

Objectives and Goals: After having completed this course, the students would be expected to understand classical concepts in the local theory of curves, surfaces and manifolds. Also the students will be familiar with the geometrical interpretation of the terminology used in the course.

Detailed Course Contents: Curves, Surfaces -Topological Invariants, Geometry on a Surface or Riemannian Geometry, Geodesics, Generalization of the Concept of Tangent and of Tangent Plane, to a Surface Manifolds -Tensor Fields - Covariant Differentiation, Tangent Vectors and Mappings, Tangent or Contravariant" Vectors, Vectors as Differential Operators, The Tangent Space to Mn at a Point, Change of Coordinates, Vector Fields and Flows on Rn, Vector Fields on Manifolds, Functionals and the Dual Space, The Differential of a Function, Scalar Products in Linear Algebra, Riemannian Manifolds and the Gradient Vector, The Tangent Bundle, The Cotangent Bundle and Phase Space, Covariant Tensors, Contravariant Tensors, Mixed Tensor, Properties, The Tensor Product of Covariant Tensors, Wedge Product, The Geometric Meaning, Special Cases, Computations and Vector Analysis, The Exterior Differential, A Coordinate Expression for d, The Pull-Back of a Covariant Tensor,. Integration of a p-Form in Rp, Integration with boundaries, Stokes' theorem, The Lie Bracket, The Lie Derivatives of Forms, Covariant Derivative, Curvature of an Affine Connection, Geodesics.

Course Outcomes: Students are expected to understand classical concepts in the local theory of curves, surfaces and manifolds. Also the students will be familiar with the geometrical interpretation of the terminology used in the course. Students will be able to apply learned concepts in other related fields.

Text Books:
T. Frankel, The Geometry of Physics, Cambridge University Press, 2012 (TB2).
A. Visconti, Introductory Differential Geometry for Physicists, World Scientific, 1992 (TB1).

Reference Books:

1. Bernard F. Schutz , Geometrical Methods of Mathematical Physics, Cambridge University Press, 1980.
2. Serge Lang, Fundamentals of Differential Geometry, Springer, 1999.

Weekly Breakdown		
Week	***Section***	***Topics***
1	1(TB2)	Curves, handouts
2	2(TB2)	Surfaces -Topological Invariants,
3	3(TB2)	Geometry on a Surface or Riemannian Geometry
4	4(TB2)	Geodesics
5	5(TB2)	Generalization of the Concept of Tangent and of Tangent Plane to a Surface
6	(TB1,TB2) 1.1a,1.2(a-c) 1.3(a-c)	Manifolds -Tensor Fields - Covariant Differentiation Tangent Vectors and Mappings, Tangent or "Contravariant" Vectors, Vectors as Differential Operators, The Tangent Space to Mn at a Point
7	(TB1)1.4(a-b)	Change of Coordinates, Vector Fields and Flows on Rn, Vector Fields on Manifolds
8	(TB1)2.1(a-d)	Functionals and the Dual Space, The Differential of a Function, Scalar Products in Linear Algebra, Riemannian Manifolds and the Gradient Vector
9	**Mid Semester Exam**	
10	(TB1)2.2a, 2.3(a-b)	The Tangent Bundle, The Cotangent Bundle and Phase Space
11	(TB1)2.4(a-e).	Covariant Tensors, Contravariant Tensors, Mixed Tensor, Properties
12	(TB1)2.5(a-e) 2.6(a-c)	The Exterior Differential, A Coordinate Expression for d,
13	(TB1)2.7a,	The Pull-Back of a Covariant Tensor,
14	3.1, 3.2, 3.3	Integration of a p-Form in Rp, Integration with boundaries, Stokes' theorem
15	(TB1)4.1, 4.2a.	The Lie Bracket, The Lie Derivatives of Forms
16	(TB1)9.1(a-c)	Covariant Derivative, Curvature of an Affine Connection Godesics
17		Review
18	**End Semester Exam**	

MATH-XXX Computational Mathematics

Credit Hours: 3-0
Prerequisite: None

Course Objectives: The main objective of this course is to train students to acquaint with the processes involved in numerical technique. The rigorous analysis of the numerical techniques to solve different problems pertaining to physical processes will be presented. Moreover, the students will get to know the programming sense of numerical procedures.
Core contents: Root finding techniques, Interpolation, Numerical differentiation, Runge-Kutta methods, Higher order method, Error control in higher order method, Conjugate gradient method, Solution of nonlinear algebraic equations, Iterative schemes, Finite difference method, Finite element method
Course Contents: Newton's method for algebraic equations, interpolation and Lagrange polynomial, Hermite polynomial, numerical differentiation, higher order Taylor methods, Runge-Kutta methods, Multistep methods, variable step size method, higher-order differential equations and system of equations, stiff differential equations, relaxation techniques for solving linear systems, the conjugate gradient method, Newton's method for nonlinear equations, the linear and nonlinear shooting methods, finite difference methods for linear and nonlinear problems, finite difference method for partial differential equations, an introduction to the finite element method

Course Outcomes: After reading this course one will be able to:
- Understand basics and advanced techniques in numerical methods
- Find solutions of linear and nonlinear algebraic equations
- Apply finite difference method for ordinary differential equations
- Overview basics of numerical method algorithm and its implementation in software (MATLAB, Maple, Python or any other software)

Textbook:
Numerical Analysis By Richard L. Burden, J. Douglas Faires and Annette M. Burden, 10 E, Cengage Learning, 2016

Reference Books

1. Applied Numerical Analysis by Curtis F. Gerald and Patrick O. Wheatley, 7th Edition, Publisher: Pearson, 2003.
2. Theory and Application of Numerical Analysis by G. M. M. Phillips and Peter J. Taylor, 2nd edition, Academic Press, 1996
3. Numerical Analysis by David Kincaid and Ward Cheney, 7th Edition, Cengage Learning, 2012

Weekly Breakdown		
Week	*Sections*	*Topic*
1	2.3, 2.4	Newton's Method and its Extensions: Newton's Method, Example 1, Convergence Using Newton's Method Error Analysis for Iterative Methods: Order of Convergence, Illustration
2	2.5, 3.1, 3.4, 4.1	Accelerating Convergence: Aitken's Method, Example 1, Definition 2.13, Theorem 2.14 Interpolation and the Lagrange Polynomial: Lagrange Interpolating Polynomial, Example 1 (overview) Hermite Interpolation: Definition 3.8, Example 1Numerical Differentiation: Example 1, Three Point Endpoint Formula, Three Point midpoint formula, Example 2, Second Derivative Midpoint Formula, Example 3
3	5.1, 5.3	The Elementary Theory of Initial-Value Problems: Definition 5.1, Example 1, Definition 5.3, Theorem 5.3, Theorem 5.4, Example 2, Example 2, Definition 5.5, Theorem 5.6, Example 3 Higher-Order Taylor Methods: Definition 5.11, Taylor method of order n, Example 1, Theorem 5.12
4	5.4	Runge-Kutta Methods: Runge-Kutta Methods of Order Two, Midpoint Method, Modified Euler Method, Example 2, Higher Order Runge-Kutta Methods, Illustration, Runge-Kutta Order Four, Example 3, Computation Comparison, Illustration
5	5.5, 5.6	Error Control and the Runge-Kutta-Fehlberg Method: Runge-Kutta-Fehlberg Method, Example 1 Multistep Methods: Definition 5.14, fourth order Adams-Bashforth Technique, Example 1, Example 2, Definition 5.15, Example 3
6	5.9, 5.10	Higher-Order Equations and Systems of Differential Equations: Definition 5.16, Theorem 517, Illustration, Higher-Order Differential Equations, Example 1 Stability: One-Step Methods, Definition 5.18, Definition 5.19, Example 1, Theorem 5.20, Example 2
7	5.11, 7.4	Stiff Differential Equations: Illustration (page 354) Relaxation Techniques for Solving Linear Systems: Definition 7.23, Example 1, Example 2
8	7.5	Error Bounds and Iterative Refinement: Example 1, Theorem 7.27, Condition number, Definition 7.28, Example 2, Illustration, iterative Refinement, Illustration, Theorem 7.29
9		**Midsemester Exam**
10	7.6	The Conjugate Gradient Method: Theorem 7.30, Theorem 7.31, Theorem 7.32, Example 1, Preconditioning, Example 2, Example 3, Illustration
11	10.1	Fixed Points for Functions of Several Variables: Example 1, Definition 10.1, Definition 10.2, Definition 10.3, Theorem 10.4, Definition 10.5, Theorem 10.6, Example 2, Accelerating Convergence
12	10.2	Newton's Method: Theorem 10.7, The Jacobian Matrix, Example 1
13	10.3, 11.1	Quasi-Newton Methods: Theorem 10.8: Sherman-Morrison Formula, Example 1 The Linear Shooting Method: Theorem 11.1, Example 1, Linear Boundary-Value Problems, Linear Shooting, Example 2
14	11.2, 11.3	The Shooting Method for Nonlinear Problems: Newton Iteration, Example 1 Finite Difference Methods for Linear Problems: Discrete Approximation, Example 1
15	11.4	Finite Difference Methods for Nonlinear Problems: Newton's Method for Iterations, Example 1
16	12.4	An Introduction to the Finite-Element Method: Defining the elements
17	12.4	Triangulating the Region, Illustration
18		**End Semester Exam**

MATH-807 Commutative Algebra

Credit Hours: 3-0
Prerequisite: Students must know the basic concepts of rings, quotient rings, integral domains and fields. Those students who have done Algebra / or equivalent will be preferred.

Course Objectives: This course aims to introduce students to the concepts of modules over commutative rings, Noetherian rings and modules, Artinian rings and valuation rings.

Detailed Course Contents: Rings, ideals, operations on ideals, radical of an ideal, nilradical, Jacobson radical, ideal quo one hour testtient, local rings, prime avoidance lemma, modules, module over commutative rings, submodules, operations on submodules, finitely generated modules, free module, annihilator of a of an element of a module, cyclic modules, quotient modules, direct sum and product of modules, module homomorphisms, short exact sequences, tensor product of modules, rings and modules of fractions, extended and contracted ideals in rings of fractions, Integral dependence, the going-up theorem, valuation rings, chain conditions, Noetherian rings and modules, Nakayama's lemma, primary decomposition, primary decomposition in Noetherian rings.

Learning Outcomes: On successful completion of this course, students will know Rings, ideals, operations on ideals, radical of an ideal, nilradical, Jacobson radical, ideal quotient, local rings, modules, submodules, operations on submodules, finitely generated modules, freemodules, direct sum and product of modules, module homomorphisms, isomorphism theorems of modules, tensor product of modules, rings and modules of fractions, Integral dependence, valuation rings, primary decomposition Noetherian rings and modules.

Recommended Books

1. M. F. Atiyah, and I. G. Macdonald, Introduction to Commutative Algebra, Addison- Wesley, 1994. ISBN: 9780201407518.
2. D. Eisenbud, "Commutative Algebra with a View Toward Algebraic Geometry", Springer, New York, 1995.
3. Thomas W. Hungerford, Algebra, Springer-Verlag, New York Inc. 1974.
4. David S Dummit, Richard M. Foote, Abstract Algebra, (3rd Ed.), 2004, John Wiley & Sons.

Weekly Breakdown	
Week	***Topics***
1	Commutative rings, integral domains, Euclidean domains, the greatest common divisor of two elements of a ring, and related theorems.
2	PID's, UFD's, and related theorems, properties of the polynomial rings, polynomial rings over fields.
3	Existence of maximal ideals of a commutative ring with unity, local rings.
4	Nilradical, Jacobson radical, related theorems, operations on ideals.
5	Radical of an ideal, ideal quotient, comaximal ideals, the Chinese Remainder Theorem.
6	Monomial ideals, operations on monomial ideals, radical of a monomial ideal, colon ideal of two monomial ideals.
7	Module over commutative rings, examples, submodules, operations on submodules.
8	Finitely generated modules, cyclic modules, Nakayama's lemma, free modules, torsion modules, torsion free modules.
9	**Mid Semester Exam**
10	Quotient modules, module homomorphisms, isomorphism theorems of modules. Direct sum and direct product of modules,
11	short exact sequences, tensor product of modules.
12	Rings and modules of fractions, localization.
13	Primary decomposition.
14	Chain conditions, Noetherian rings, and modules
15	Artinian rings and modules.
16	Primary decomposition in Noetherian rings.
17	Review
18	**End Semester Exam**

MATH-817 Advanced Functional Analysis

Credit Hours: 3+0
Prerequisite: None

Course Objectives: This course presents functional analysis from a more advanced perspective. The main objective are to 1) understand the classic results of Functional Analysisincluding Zorn's Lemma and Hahn-Banach Theorem, 2) understand the basic concepts of Fixed Point Theory, 3) know and understand the topics on approximation theory.

Previous Knowledge: A student who wishes to opt this course is recommended to have a previous knowledge of elementary functional analysis including Metric Spaces, Normed Spaces, Banach Spaces, Inner Product Spaces and Hilbert spaces. Furthermore, student is required to have a good command on elementary linear algebra.

Core Contents: Fundamental Theorems for Normed and Banach Spaces, Banach Fixed Point Theorem and its applications, Applications of Banach Fixed Point Theorem, Approximation Theory.

Detailed Course Contents: Fundamental Theorems for Normed and Banach Spaces: Zorn's Lemma, Hahn-Banach Theorem, Hahn-Banach Theorem for Complex Vector Spaces and Normed Spaces, Adjoint Operator, Reflexive Spaces, Category Theorem, Uniform Boundedness Theorem, Strong and Weak Convergence, Convergence of Sequences of Operators and Functionals, Open Mapping Theorem, Closed Linear Operators. Closed Graph Theorem.
Further Applications: Banach Fixed Point Theorem: Banach Fixed Point Theorem, Application of Banach's Theorem to Linear Equations, Applications of Banach's Theorem to Differential Equations, Application of Banach's Theorem to Integral Equations.
Approximation Theory: Approximation in Normed Spaces, Uniqueness, Strict Convexity, Uniform Approximation, Chebyshev Polynomials, Approximation in Hilbert Space

Course Outcomes: This course is specially designed for students who want to choose functional analysis and fixed point theory as their specialty. On successful completion of this course, the students will:

- Be able to work with fundamental concepts in functional analysis.
- Have a grasp of formal definitions and rigorous proofs in functional analysis.
- Be able to apply abstract ideas to concrete problems in analysis.
- Be aware of applications of basic techniques and theorems of functional analysis in other areas of mathematics, such as fixed point theory, approximation theory, and the theory of ordinary differential equations.

Text Book: Erwin Kreyszig, Introductory Functional Analysis with Applications, Wiley; First edition 1989.

Reference Books:

1. J. B. Conway. A Course in Functional Analysis. Springer-Verlag , New York, 1985.
2. George Bachman, Lawrence Narici, Functional Analysis, Dover Publications; 2nd edition, 1998.

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1, 1.3, 1.4, 1.6	**Review:** Metric spaces, Open set, Closed set, Cauchy sequence, Complete metric spaces,
2	2.2,3.1,3.2	**Review:** Normed spaces, Banach spaces, Inner product spaces, Hilbert spaces
3	4.1,4.2	Zorn's Lemma, Hahn-Banach Theorem
4	4.3, 4.5	Hahn- Banach Theorem for complex vector spaces and Normed Spaces, Adjoint Operator
5	4.6	Reflexive spaces
6	4.7	Category Theorem, Uniform Boundedness Theorem
7	4.8	Strong and Weak Convergence
8	**Mid Semester Exam**	
9	4.9	Convergence of sequences of Operators and functionals
10	4.12	Open Mapping Theorem
11	4.13	Closed Linear Operators, Closed Graph Theorem
12	5.1	Banach Fixed Point Theorem
13	5.2, 5.3	Applications of Banach's Theorem to Linear Equations and Differential Equations
14	5.4	Applications of Banach's Theorem to Integral Equations
15	6.1, 6.2	Approximation I Normed Spaces, Uniqueness, Strict Convexity
16	6.3	Uniform Approximation, Chebyshev Polynomial
17		Review
18	**End Semester Exam**	

MATH-818 Theory of Ordinary Differential Equations

Credit Hours: 3-0
Prerequisite: None

Course Objectives: Modern technology requires a deeper knowledge of the behavior of real physical phenomena. Mathematical models of real-world phenomenon are formulated as algebraic, differential or integral equations (or a combination of them). After the construction of equations, the study of their properties is necessary. At this stage the theory of ordinary differential equations plays a significant role. In this course we shall discuss the stability theory and phase-plane analysis of dynamical systems, bifurcation theory, Non-oscillation and oscillation theory and the existence theory of differential equations.

Detailed Course Contents: General theory of linear equations, Homogeneous Linear Equations with periodic coefficients: Floquet multipliers, Floquet Theorem, Stability of linear equations, Stability of linear equations by Lozinskii measures, Perturbations of linear equations, Lyapunov function method for autonomous equations, Lyapunov function method for nonautonomous equations, General theory of autonomous equations, Poincar´e– Bendixson Theorem, Periodic solutions and orbital stability, Basic concepts of bifurcation theory, One-dimensional bifurcations for scalar equations, One-dimensional bifurcations for planar systems, Hopf bifurcations for planar systems, Second-order linear equations, Self- adjoint second-order differential equation, Cauchy Function and Variation of Constants Formula, Sturm-Liouville problems, Zeros of solutions and disconjugacy, Factorizations and recessive and dominant solutions Oscillation and non-oscillation, Applications of the CMT to BVPs, Lower and upper solutions, Nagumo condition, Lipschitz condition and Picard- Lindelof Theorem, Equicontinuity and the Ascoli-Arzela Theorem, Cauchy-Peano Theorem, Extendability of solutions, Basic Convergence Theorem, Continuity of solutions with Respect to ICs, Kneser's Theorem. Differentiating solutions with respect to ICs, Maximum and minimum solutions.

Course Outcomes: Students are expected to understand topics such as stability theory, bifurcation theory, phase-plane analysis of dynamical systems, and existence theory of differential equations.

Text Books:
Theory of Differential Equations, W. G. Kelley, A. C. Peterson Springer, 2010.
Qingkai Kong, A Short Course in Ordinary Differential Equations, Springer 2014 (Referred as QK)

Reference Books:

1. Ordinary differential equations by I.G.Petrovski, Dover Publications, Inc., 1973
2. Theory of ordinary differential equations, Coddington E.A. and Levinson, New York: McGraw-Hill, 1955.
3. Nonlinear Ordinary Differential Equations, D. W. Jordan and P. Smith, Oxford University Press, 2007

Weekly Breakdown		
Week	***Section***	***Topics***
1	Ch. 1	First order equations: Existence, Bifurcation, Stability
2	2.3	The Matrix Exponential Function, Putzer Algorithm, Lozinski measure
3	2.5	Homogeneous Linear Equations with periodic coefficients: Floquet multipliers, Floquet Theorem
4	3.1-3.3	Phase plane diagram, homoclinic orbits, Hamiltonian systems
5	3.4, 3.5	Stability of nonlinear systems, Semi-group property, Lyapunov function method for autonomous and non-autonomous equations, Linearization of nonlinear systems
6	3.6	Existence and nonexistence of periodic, Solutions, Poincare–Bendixson Theorem, Bendixson-Dulac Theorem, Lienard's Theorem
7	3.7	Three-dimensional systems
8	5.1, 5.2 (QK)	Basic concepts of bifurcation theory, One-dimensional bifurcations for scalar equations
9	**Mid Semester Exam**	
10	5.3, 5.4 (QK)	One-dimensional bifurcations for planar systems, Hopf bifurcations for planar systems
11	5.1,5.2	Self-adjoint second-order differential equation: Basic concepts
12	5.3	Cauchy Function and variation of constants formula
13	5.4	Sturm-Liouville problems
14	5.5	Zeros of solutions and disconjugacy
15	5.6 ,5.7	Factorizations and recessive and dominant solutions, The Riccati Equation,
16	5.9	Green Function, Contraction Mapping Theorem (handouts)
17		Review
18	**End Semester Exam**	

MATH-819 Analysis of Fractional Differential Equations

Credit Hours: 3-0
Prerequisite: None

Objectives and Goals: The aim of the course is to motivate students to study different topics of the theory of fractional calculus and fractional differential equations.

Core Contents: BVPs for Nonlinear Second-Order ODEs, Existence and Uniqueness Theorems, Riemann–Liouville Differential and Integral Operators, Riemann–Liouville Differential and Integral Operators, Caputo's Approach, Mittag-Leffler Functions, Existence and Uniqueness Results for Riemann–Liouville and Caputo Fractional Differential Equations

Detailed Course Contents: BVPs for Nonlinear Second-Order ODEs: Contraction Mapping Theorem, Application of the Contraction Mapping Theorem to a Forced Equation Application of Contraction Mapping Theorem to BVPs, Lower and Upper Solutions,Nagumo Condition. Existence and Uniqueness Theorems: Basic Results, Lipschitz Condition and Picard-Lindelof Theorem, Equicontinuity and the Ascoli-Arzela Theorem, Cauchy-Peano Theorem, Extendability of Solutions, Basic Convergence Theorem, Continuity of Solutions with Respect to ICs, Kneser's Theorem, Differentiating Solutions with Respect to ICs, Maximum and Minimum Solutions.
Introduction to Fractional Calculus: Motivation, The Basic Idea, An Example Application of Fractional Calculus. Riemann–Liouville Differential and Integral Operators: Riemann–Liouville Integrals, Riemann–Liouville Derivatives, Relations Between Riemann–Liouville Integrals and Derivatives, Grunwald–Letnikov Operators. Caputo's Approach: Definition and Basic Properties, Nonclassical Representations of Caputo Operators.
Mittag-Leffler Functions: Definition and Basic Properties.
Existence and Uniqueness Results for Riemann–Liouville Fractional Differential Equations. Single-Term Caputo Fractional Differential Equations: Existence of Solutions, Uniqueness of Solutions, Influence of Perturbed Data, Smoothness of the Solutions, Boundary Value Problems. Advanced Results for Special Cases: Initial Value Problems for Linear Equations,Boundary Value Problems for Linear Equations, Stability of Fractional DifferentialEquations.

Course Outcomes: Students are expected to understand:

- Existence theory for second order ordinary differential equations
- Properties of fractional operators
- Existence theory of fractional differential equations

Text Books:
Walter G. Kelley, Allan C. Peterson, Theory of Differential Equations, Second Edition, Springer, (2010) (Referred as KP).
Kai Diethelm, The Analysis of Fractional Differential Equations, Springer, (2010)(Referred as KD).

Reference Books:

1. Podlubny, Fractional Differential Equations. Academic Press, San Diego (1999).
2. R. Hilfer, Applications of Fractional Calculus in Physics, World Scientific Publishing (2000).
3. A.A. Kilbas, H.M. Srivastava, J.J. Trujillo, Theory and applications of fractional differential equations, vol 204. North-Holland mathematics studies. Elsevier, Amsterdam (2006).

Weekly Breakdown

Week	*Section*	*Topics*
1	7.1,7.2 (KP)	BVPs for Nonlinear Second-Order ODEs: Contraction Mapping Theorem, Application of the Contraction Mapping Theorem to a Forced Equation.
2	7.3-7.5	Application of Contraction Mapping Theorem to BVPs, Lower and Upper Solutions, Nagumo Condition.
3	8.1-8.3	Existence and Uniqueness Theorems: Basic Results, Lipschitz Condition and Picard-Lindelof Theorem, Equicontinuity and the Ascoli-Arzela Theorem.
4	8.4-8.6	Cauchy-Peano Theorem, Extendability of Solutions, Basic Convergence Theorem
5	8.7-8.10	Continuity of Solutions with Respect to ICs, Kneser's Theorem, Differentiating Solutions with Respect to ICs, Maximum and Minimum Solutions.
6	1.1-1.3 (KD)	Introduction to Fractional Calculus: Motivation, The Basic Idea, An Example Application of Fractional Calculus.
7	2.1, 2.2	Riemann–Liouville Differential and Integral Operators: Riemann–Liouville Integrals, Riemann–Liouville Derivatives.
8	2.3, 2.4	Relations Between Riemann–Liouville Integrals and Derivatives, Grünwald–Letnikov Operators.
9	**Mid Semester Exam**	
10	3.1, 3.2	Caputo's Approach: Definition and Basic Properties, Nonclassical Representations of Caputo Operators.
11	4	Mittag-Leffler Functions: Definition and Basic Properties.
12	5	Existence and Uniqueness Results for Riemann–Liouville Fractional Differential Equations.
13	6.1, 6.2	Single-Term Caputo Fractional Differential Equations: Basic Theory and Fundamental Results: Existence of Solutions, Uniqueness of Solutions.
14	6.3, 6.4	Influence of Perturbed Data, Smoothness of the Solutions
15	6.5	Boundary Value Problems.
16	7.1-7.3	Advanced Results for Special Cases: Initial Value Problems for Linear Equations, Boundary Value Problems for Linear Equations, Stability of Fractional Differential Equations.
17		Review
18	**End Semester Exam**	

MATH-820 Calculus of Variations and Optimal Control

Credit Hours: 3-0
Prerequisite: None

Course Objectives: The major purpose of this course is to present theoretical ideas and analytic and numerical methods to enable the students to understand and efficiently solve optimization problems.

Core Contents: The Finite dimensional problem: The free problem. Equality constrained problem. The inequality constrained problem, Newton's Method. The basic theory of the calculus of variations: Introduction, Some examples. Critical point conditions. Additional necessary conditions. Miscellaneous results. Sufficiency theory. Several dependent variables. Optimal control, The minimal time problem, Unconstrained Reformulations. Constrained calculus of variations problems. Kuhn-Tucker reformulation. Numerical methods and results.Kuhn-Tucker method. Introduction to fractional calculus. Fractional calculus of variations, Fractional Euler–Lagrange equations

Course Outcomes: Students are expected to understand:

- The theory of the calculus of variations.
- The optimal control problems.
- Numerical methods and results for optimization.
- Fractional calculus of variations.

Text Book:
John Gregory, Cantian Lin, Constrained Optimization in the Calculus of Variations and Optimal Control Theory, Springer (1992).
Ricardo Almeida, Dina Tavares Delfim F. M. Torres, (RAD) The Variable-Order Fractional Calculus of Variations, Springer 2019.

Reference Books:
1. M. D. Intriligator, Mathematical Optimization and Economic Theory, Siam (2002).
2. Pablo Pedregal, Optimization and Approximation, Springer (2017)
3. Daniel Liberzon, Calculus of Variations and Optimal Control Theory, PRINCETON UNIVERSITY PRESS, (2012).

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1,1.2	The Finite dimensional problem: The free problem, The equality constrained problem.
2	1.3, 1.4	The inequality constrained problem, Newton's Method.
3	2.1-2.3	The basic theory of the calculus of variations: Introduction, Some examples
4	2.3	Critical point conditions.
5	2.4, 3.1	Additional necessary conditions, Miscellaneous results
6	3.2	Sufficiency theory.
7	3.3	Several dependent variables.
8	4.1	Optimal control: A basic problem
9	**Mid Semester Exam**	
10	4.2, 5.1	The minimal time problem: An example of abnormality. Unconstrained Reformulations: The optimal control problems.
11	5.2,5.3	Constrained calculus of variations problems, Kuhn-Tucker reformulation
12	6.1	Numerical methods and results: The basic Problem in calculus of variations
13	6.2	Numerical transversality conditions for general problems
14	6.3	Kuhn-Tucker method
15	2.1,2.2 (RAD)	Introduction to fractional calculus
16	3.2	Fractional calculus of variations, Fractional Euler–Lagrange equations
17		Review
18	**End Semester Exam**	

MATH-821 Analytical Approximate Solutions of ODEs

Credit Hours: 3-0
Prerequisite: None

Course Objectives: The objective of this course is to introduce analytical and approximate methods for differential equations and make students familiar with advanced topics in spectral methods.

Core Contents: The variational iteration method, The Adomian decomposition method, Perturbation method, Hamiltonian approach, Homotopy analysis method, spectral methods, Fourier and Chebyshev Series, Discrete least square approximation, Chebyshev interpolation, Tau-spectral method. Collocation spectral methods.

Detailed Course Contents: The variational iteration method: Application of the variational iteration method. The Adomian decomposition method: Application of the Adomian decomposition method. Perturbation method: Theoretical background, application of the perturbation method. Energy balance method: Theoretical background, application of the energy balance method. Hamiltonian approach: Theoretical background, application of the Hamiltonian approach. Homotopy analysis method: Theoretical background. Homotopy analysis method: application of the homotopy analysis method.
Fourier and Chebyshev Series, The trigonometric Fourier series. The Chebyshev series. Discrete least square approximation. Chebyshev discrete least square approximation. Orthogonal polynomials least square approximation. Orthogonal polynomials and Gauss-type quadrature formulas. Chebyshev projection. Chebyshev interpolation. Collocation derivative operator. General formulation for linear problems. Tau-spectral method. Collocation spectral methods: A class of nonlinear boundary value problems. Spectral-Galerkinmethods.

Learning Outcomes: On successful completion of this course students will be able to:

- Understand and apply approximate methods such as the variational iteration method,
- The Adomian decomposition method, Perturbation method, Hamiltonian approach, Homotopy analysis method
- Understand and apply spectral methods for solving differential equations.

Textbooks:
M. Hermann, M. Saravi, (HS) Nonlinear Ordinary Differential Equations, Analytical Approximations and Numerical Methods, Springer (2016)
C. I. Gheorghiu, (CIG) Spectral Methods for Differential Problems, TiberiuPopoviciu Institute of Numerical Analysis (2007)

Reference Book:

1. C. Canuto, M. Y. Hussaini, A. Quarteroni and T. A. Zang, Spectral Methods: Fundamentals in Single Domains, Springer (2006)
2. Lloyd N. Trefethen, Approximation Theory and Approximation Practice, Siam (2013).

Weekly Breakdown		
Week	***Section***	***Topics***
1	HS 2.1-2.3	The variational iteration method, application of the variational iteration method.
2	2.4, 2.5	The Adomian decomposition method, application of the Adomian decomposition method.
3	3.1	Perturbation method: theoretical background, application of perturbation method.
4	3.2	Energy balance method: theoretical background, application of energy balance method.
5	3.3	Hamiltonian approach: theoretical background, application of the Hamiltonian approach.
6	3.4	Homotopy analysis method: theoretical background.
7	3.4 (cont.)	Homotopy analysis method: application of the homotopy analysis method.
8	1.1,1.2. 1,1.2.2	General properties, Fourier and Chebyshev Series, The trigonometric Fourier series, The Chebyshev series.
9	**Mid Semester Exam**	
10	1.2.3	Discrete least square approximation.
11	1.2.4,1. 2.6	Chebyshev discrete least square approximation, Orthogonal polynomials least square approximation, Orthogonal polynomials and Gauss-type quadrature formulas
12	1.3,1.4	Chebyshev projection, Chebyshev interpolation.
13	1.4 (cont.)2.1	Chebyshev interpolation (cont.) Collocation derivative operator. The idea behind the spectral methods.
14	2.2,2.3	General formulation for linear problems, Tau-spectral method.
15	2.4	Collocation spectralmethods (pseudospectral), A class of nonlinear boundary value problems.
16	2.5	Spectral-Galerkinmethods.
17		Review
18	**End Semester Exam**	

MATH-822 Mathematical Modelling-I

Credit Hours: 3-0
Prerequisite: None

Course Objectives: The course focuses on the application of "dimensional methods" tofacilitate the design and testing of engineering problems. It aims to develop a practical approach to modeling and dimensional analysis. This course will be well received and will prove to be an invaluable reference to researchers and students with an interest dimensional analysis and modeling and those who are engaged in design, testing and performancesevaluation of engineering and physical system.

Core Contents: The course includes the theory of matrix algebra and linear algebra, thetheory of dimension, transformation of dimensions and structure of physical variables, dimensional similarities and models law. This course will cover the nature of dimensional analysis use in mathematical modeling.

Detailed Course Contents: Mathematical Preliminaries, Matrices and Determinants, Operation with Matrices, The rank of matrices and Systems of linear equations, Formats and Classification, Numerical, Symbolic and Mixed format, Classification of Physical Quantities, dimensional system, General Statement, The SI system, Structure, Fundamental dimension, Derived dimensional units with and without specific names, Rules of etiquettes in Writing dimensions.

Other than SI dimensional systems, A note on the classification of dimensional systems, Transformation of Dimensions, Numerical equivalences, Techniques, Examples, Problems, Arithmetic of Dimensions, Dimensional Homogeneity.

Equations, graphs, Problems, Structure of Physical Relations, the dimensional matrix, Number of independent sets of products of given dimension 1,11, Special case, Buckingham's theorem, Selectable and non-selectable dimensions, Minimum number of independent product of variables of given dimension, Constancy of the sole dimensionless product, Number of dimension equals or exceeds the number of variables, Systematic determination of Complete Set of Products of Variable Transformations, Theorems related to some specific transformations, Transformations between systems of different d matrices, Number of Sets of Dimensionless Products of Variables

Distinct and equivalent sets, Changes in dimensional set not affecting the dimensional variables, Prohibited changes in dimensional set.

Relevancy of Variables, Dimensional irrelevancy, Condition, Adding a dimensionally irrelevant variables to a set of relevant variables, Physical irrelevancy, Problems, Economy of Graphical Presentation, Number of curves and charts, Problems, Forms of Dimensionless Relations

General classification, Monomial is Mandatory, Monomial is impossible, Reconstructions, Sequence of Variables in the Dimensional Set, Dimensionless physical variable is present, Physical variables of identical dimensions are present, Independent and dependent variables.

Learning Outcomes: Students are expected to understand Fundamentals dimension of dimensional analysis.

Text Books:

Thomas Szitres, Applied Dimensional Analysis and Modeling, Elsevier Inc., 2007. (Referred as TS).

S.H. Friedberg, A.J. Insel, L.E.Spence, Linear Algebra, Prentice-Hall, Inc., Englewood Cliffs, N.J. USA,979 (referred as FIS

Weekly Breakdown		
Week	***Section***	***Topics***
1	TS, Ch. 1, FIS, Ch. 3	Mathematical Preliminaries, Matrices and Determinants, Operation with Matrices, The rank of matrices and Systems of linear equations
2	TS Chs. 2, 3	Formats and Classification, Numerical, Symbolic and Mixed format Classification of Physical Quantities, dimensional system, General Statement, The SIsystem
3	Ch 3	Structure, Fundamental dimension, Derived dimensional units with and withoutspecific names, Rules of etiquettes in Writing dimensions Other than SI dimensional systems
4	Chs 3,4	A note on the classification of dimensional systems, Transformation of Dimensions,Numerical equivalences, Techniques, Examples, Problems
5	Chs 5, 6	Arithmetic of Dimensions,Dimensional Homogeneity
6	Chs 6,, 7	Equations, graphs, Problems, Structure of Physical Relations, the dimensional matrix, Number of independent sets of products of given dimension 1,11, Special case
7	Ch 7	Buckingham's theorem, Selectable and non selectable dimensions, Minimum number of independent product of variables of given dimension, Constancyof the sole dimensionless product
8	Chs 7,8	Number of dimension equals or exceeds the number of variables Systematic determination of Complete Set of Products of Variable
9	**Mid Semester Exam**	
10	Ch 9	Transformations, Theorems related to some specific transformations, Transformations between systems of different d matrices
11	Ch 10	Number of Sets of Dimensionless Products of Variables Distinct and equivalent sets, Changes in dimensional set not affecting the dimensionalvariables, Prohibited changes in dimensional set
12	Ch 11	Relevancy of Variables, Dimensional irrelevancy, Condition, Adding a dimensionally irrelevant variables to a set of relevant variables,
13	Chs 11, 12	Physical irrelevancy, Problems, Economy of Graphical Presentation Number of curves and charts, Problems
14	Ch 13	Forms of Dimensionless Relations, General classification, Monomial is Mandatory,Monomial is impossible,Reconstructions
15	Ch 14	Sequence of Variables in the Dimensional Set, Dimensionless physical variable is present,
16	Ch 14	Physical variables of identical dimensions are present, Independent and dependent variables
17		Review of Material
18	**End Semester Exam**	

MATH-XXX Advanced Topology

Credit Hours: 3-0
Prerequisite: Nil

Course Objectives: The course aims at developing an understanding about advanced concepts of Topology which are the basic tools of working mathematicians in a variety of fields. It covers some cover concepts including compactness and connectedness and explains how these concepts of Analysis are generalized to Topology.

Core Contents: Topological Spaces, Neighborhood, Bases, Initial & Final Topology, Quotient Spaces, Inadequacy of Sequences, Nets, Filters & Ultra Filters, Lindelöf Spaces, Separable Spaces, Compactness, Compactness in terms of filters, Locally Compact Spaces, One-point compactification, Stone-Cech Compactification, Para-compactness, Connectedness, Connected Components, Pathwise & Locally Connected Spaces,

Detailed Course Contents: Topological Spaces, Neighborhood, Neighborhood base, Subbases, Local Bases, Bases, Initial/Weak Topology and its Applications, Final/Strong Topology and its Applications, Quotient Spaces, Inadequacy of Sequences, Nets and their properties, Filters, Filter bases, Ultra Filters, Topology induced by filters, Relation b/w filters & Nets, Lindelöf Spaces, Separable Spaces, Compactness, Compactness in terms of Closedness & filters, Countable compactness, Limit-point compactness, One-point compactification, Stone-Cech compactifications, Connectedness, Connected components, Totally Disconnected spaces, Locally connected spaces and its applications, Pathwise connectedness and its relation to connectedness.

Course Outcomes: Upon completion of this course, the student should be able to:

- Continuous mappings, Disjoint Homeomorphism, Weak and Strong topologies, Quotient spaces
- Inadequacy of Sequences, Nets, Filters & Ultra Filters
- Lindelöf Spaces, Separable Spaces
- Compactness, Countable, Limit-point and local compactness
- One-point & Stone-Cech Compactifications
- Connectedness, Connected components, Totally disconnectedness, Pathwise & Local Connectedness

Text Book: S. Willard, "General Topology", Dover Publications; Illustrated Edition, (2004)

Reference Books:

1. James R. Munkres, "Topology", Prentice, Hall, Inc., 2nd Edition (2000)
2. T. D. Bradley, T. Bryson, J. Terilla, "Topology: A Categorical Approach", MIT Press, (2020)
3. G. Preuss, "Foundations of Topology: An Approach to Convenient Topology", Springer, 2nd Edition, (2002).
4. J. Kelly, "General Topology", Springer, (2005).

Weekly Breakdown		
Week	*Section*	*Topics*
1	Sec. 3-4	Review of Topological spaces and Examples, Neighborhood operators, Topology induced by neighborhoods, Neighborhood bases, Open, closed, interiors and closures in terms of neighborhoods
2	Sec. 5-6	Subbases, Bases, Local bases and their properties, Subspaces and its properties, and related results
3	Sec. 7	Continuous functions between topologies, and their characterizations using neighborhood operators, characterizations of spaces using continuous mappings, Continuous functions to and from a plane., Disjoint homeomorphisms
4	Sec.8	Weak Topologies and their applications, Box products and their related results, Tychonoff Topologies
5	Sec. 9	Strong/Final Topologies and their applications, Quotient spaces, identification spaces, Quotients vs Decompositions
6	Sec. 10	Inadequacy of sequences, sequentially convergences, 1st, and 2nd countable and its applications
7	Sec. 11	Nets, Ultra nets and their examples, subnets and related results, Net convergence in topologies
8	Sec. 12	Filters, Ultrafilters, Topologies induced by filters, Filter convergence in topological spaces, Relationship between filters and nets
9	**Mid Semester Exam**	
10	Sec.13-14	Lower Separation axioms and related results, Regular and completely regular spaces
11	Sec. 15-16	Normal spaces and related results, Urysohn Lemma and Tietze Extension Theorem, Shrinkable spaces, Separable and Lindelöf spaces and Results
12	Sec. 17	Compactness, Compactness in terms of neighborhoods and filters, sequentially compactness and their related results, Countable compactness, and related theorems
13	Sec. 18	Locally compact spaces, examples and its relations with compactness, countable compactness and sequentially compactness, and their related results
14	Sec. 19	Compactifications, Alexandroff Compactifications, Stone-Cech Compactifications
15	Sec. 26	Connectedness and examples, Connectedness in terms of neighborhood and filters, Mutual Separated spaces, Connected components and their related results
16	Sec. 27	Pathwise connectedness and locally connectedness, examples and their related results and their relation with connectedness and mutual separateness
17	Sec. 29	Totally disconnected spaces, examples and related results, Zero-dimensional spaces, examples, and related theorems.
18	**End Semester Exam**	

MATH-903 Partial Differential Equations-I

Credit hours: 3-0
Prerequisite: None

Course Objectives: Modern technology requires a deeper knowledge of the behavior of real physical phenomena. Mathematical models of real world phenomenon are formulated as algebraic, differential or integral equations (or a combination of them). After the construction of equations the study of their properties is necessary. At this stage the theory of ordinary differential equations plays a significant role. Partial Differential Equations (PDEs) are at the heart of applied mathematics and many other scientific disciplines. PDEs are at the heart of many scientific advances. The behavior of many material object in nature, with time scales ranging from picoseconds to millennia and length scales ranging from sub-atomic to astronomical, can be modelled by PDEs or by equations with similar features. Indeed, many subjects revolve entirely around their underlying PDEs. The role of PDEs within mathematicsand in other sciences is fundamental and is becoming increasingly significant. At the same time, the demands of applications have led to important developments in the analysis of PDEs, which have in turn proved valuable for further different applications. The goal of the course is to provide an understanding of, and methods of solution for, the most important types of partial differential equations that arise in Mathematical Physics. Advanced topics such as week solutions and discontinuous solutions of nonlinear conservation laws are also considered.

Detailed Course Contents: First-order Partial Differential Equations: Linear First-order Equations, The Cauchy Problem for First-order Quasi-linear Equations, Fully-nonlinear First- order Equations, General Solutions of Quasi-linear Equations. Second-order Partial Differential Equations: Classification and Canonical Forms of Equations in Two Independent Variables, Classification of Almost-linear Equations in Rn. One Dimensional Wave Equation: The Wave Equation on the Whole Line. D' Alembert Formula, The Wave Equation on the Half-line, Reflection Method. Mixed Problem for the Wave Equation, Inhomogeneous Wave Equation, and Conservation of the Energy. One Dimensional Diffusion Equation: The Diffusion Equation on the Whole Line, Diffusion on the Half-line, Inhomogeneous Diffusion Equation on the Whole Line, Maximum- minimum Principle for the Diffusion Equation. Weak Solutions, Shock Waves and Conservation Laws: Weak Derivatives and Weak Solutions Conservation Laws, Burgers' Equation, Weak Solutions. Riemann Problem, Discontinuous Solutions of Conservation Laws, Rankine-Hugoniot Condition. The Laplace Equation: Harmonic Functions. Maximum-minimum Principle, Green's Identities, Green's Functions, Green's Functions for a Half-space and Sphere, Harnack's Inequalities and Theorems.

Course Outcomes: Students are expected to understand topics such as first and second order linear classical PDEs as well as nonlinear equations. Explicate formulae and derive properties of solutions for problems with homogenous and inhomogeneous equations; without boundaries and with boundaries.

Text Books: Ioannis P Stavroulakis, Stepan A Tersian, Partial Differential Equations: An Introduction with Mathematica and Maple, World Scientific, 2004.

Reference Books:

1. Walter A Strauss, Partial Differential Equations: An introduction, John Wiley & Sons, 2008.
2. Peter J. Olver, Introduction to Partial Differential Equations, Springer, 2014.

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1-1.2	Introduction to partial differential equations, Linear First-order Equations.
2	1.3	The Cauchy Problem for First-order Quasi-linear Equations. Existence and blowup of solution.
3	1.4	Quasi-linear Equations: theory and methods of general solution.
4	Handouts	Classification of system of partial differential equations. Method of solutions for system of partial differential equations.
5	1.5	Fully-nonlinear First-order Equations: Theory and methods of solution.
6	2.1, 2.2	Methods of solution for Linear Equations. Classification and Canonical Forms of Equations in two Independent Variables.
7	3.1, 3.2	The Wave Equation on the Whole Line. D'Alembert Solution, The Wave Equation on the Half-line.
8	3.3	Reflection Method, Mixed Problem for the Wave Equation.
9	**Mid Semester Exam**	
10	3.4	Inhomogeneous Wave Equation.
11	3.5	Conservation of the Energy.
12	4.1	Maximum-minimum Principle for the Diffusion Equation
13	4.2	The Diffusion Equation on the Whole Line.
14	4.3, 4.4	Diffusion on the Half-line. Inhomogeneous Diffusion Equation on the Whole Line.
15	5.1,5.2	Weak Derivatives and Weak Solutions, Conservation Laws.
16	5.3,5.4	Burgers' Equation, Weak Solutions. Riemann Problem.
17		Review
18	**End Semester Exam**	

MATH-904 Partial Differential Equations-II

Credit Hours: 3-0
Prerequisite: MATH 903 Partial Differential Equations-I

Course Objectives: The aim of the course is to motivate students to study different topics of the theory of partial differential equations. The course provides an overview on different topics of the theory of partial differential equations, Wave Equation—Properties of Solutions, The Notion of Energy of Solutions, Phase Space Analysis for the Heat Equation, Phase Space Analysis for Wave Models.

Core Contents: Basics for Partial Differential Equations, The Cauchy-Kovalevskaja Theorem, Holmgren's Uniqueness Theorem, Method of Characteristics, Burgers' Equation, Laplace Equation-Properties of Solutions. Heat Equation—Properties of Solutions.

Detailed Course Contents: Classification of Linear Partial Differential Equations of Kovalevskian Type, Classification of Linear Partial Differential Equations of Second Order, Classification of Linear Systems of Partial Differential Equations, Classification of Domains and Statement of Problems, Classification of Solutions. Classical Version, Abstract Version, Applications of the Abstract Cauchy-Kovalevskaja Theorem. Classical Version, Abstract Version, Quasilinear Partial Differential Equations of First Order, The Notion of Characteristics: Relation to Systems of Ordinary Differential Equations. Influence of the Initial Condition, Application of the Inverse Function Theorem. Classical Burgers' Equation, Other Models Related to Burgers' Equation. Poisson Integral Formula, Properties of Harmonic Functions, Other Properties of Elliptic Operators or Elliptic Equations. Boundary Value Problems of Potential Theory, Potential Theory and Representation Formula, Maximum-Minimum Principle. Qualitative Properties of Solutions of the Cauchy Problem for the Heat Equation, Mixed Problems for the Heat Equation d'Alembert's Representation in R, Wave Models with Sources or Sinks, Kirchhoff's Representations. Propagation of Singularities. Energies for Solutions to the Wave Equation, Examples of Energies for Other Models**.** Behavior of Local Energies. The Classical Heat Equation, The Classical Heat Equation with Mass, The Classical Wave Model, The Classical Damped Wave Model, Viscoelastic Damped WaveModel, Klein-Gordon Model

Course Outcomes: Students are expected to understand different topics of the theory of partial differential equations such as: The Cauchy-Kovalevskaja Theorem, Holmgren's Uniqueness Theorem, Method of Characteristics, Burgers' Equation, Laplace Equation- Properties of Solutions. Heat Equation—Properties of Solutions.

Text Book: Marcelo R. Ebert, Michael Reissig, Methods for Partial Differential Equations, Springer International Publishing AG 2018.

Reference Books:

1. Fritz John, Partial Differential Equations, Springer-Verlag, 1978.
2. Michael Eugene Taylor, Partial Differential Equations I: Basic Theory, Springer- Verlag, 1996.
3. R.C. McOwen, Partial Differential Equations: Methods and Applications, Pearson, 2004.

Weekly Breakdown		
Week	***Section***	***Topics***
1	3.1-3.5	Basics for Partial Differential Equations: Classification of Linear Partial Differential Equations of Kovalevskian Type, Classification of Linear Partial Differential Equationsof Second Order, Classification of Linear Systems of Partial Differential Equations, Classification of Domains and Statement of Problems, Classification of Solutions.
2	4.1-4.3	The Cauchy-Kovalevskaja Theorem: Classical Version, Abstract Version, Applications of the Abstract Cauchy-Kovalevskaja Theorem.
3	5.1,5.2	Holmgren's Uniqueness Theorem: Classical Version, Abstract Version,
4	6.1,6.2	Method of Characteristics: Quasilinear Partial Differential Equations of First Order, The Notion of Characteristics: Relation to Systems of Ordinary Differential Equations.
5	6.3, 6.4	Influence of the Initial Condition, Application of the Inverse Function Theorem.
6	7.1,7.2	Burgers' Equation: Classical Burgers' Equation, Other Models Related to Burgers' Equation.
7	8.1-8.3	Laplace Equation-Properties of Solutions: Poisson Integral Formula, Properties of Harmonic Functions, Other Properties of Elliptic Operators or Elliptic Equations.
8	8.4, 9.1,9.2	Boundary Value Problems of Potential Theory, Heat Equation-Properties of Solutions: Potential Theory and Representation Formula, Maximum-Minimum Principle.
9	**Mid Semester Exam**	
10	9.3, 9.4	Qualitative Properties of Solutions of the Cauchy Problem for the Heat Equation, Mixed Problems for the Heat Equation
11	10.1, 10.4	Wave Equation—Properties of Solutions: d'Alembert's Representation in R, Wave Models with Sources or Sinks, Kirchhoff's Representations.
12	10.6, 11.1, 11.2	Propagation of Singularities. The Notion of Energy of Solutions: Energies for Solutions to the Wave Equation, Examples of Energies for other Models.
13	11.3, 12.1,12.2	Behavior of Local Energies. Phase Space Analysis for the Heat Equation: The Classical Heat Equation, The Classical Heat Equation with Mass.
14	14.1, 14.2	Phase Space Analysis for Wave Models: The Classical Wave Model, The Classical Damped Wave Model,
15	14.3	Viscoelastic Damped Wave Model
16	14.4	Klein-Gordon Model
17	-	Review
18	**End Semester Exam**	

MATH-905 Symmetry Methods for Differential Equations-I

Credit Hours: 3-0
Prerequisites: None

Course Objectives: This lecture course aims to introduce students to the basic concepts of Symmetry Methods. Whereas there are standard techniques for solving differential equations, apart from the first order equations there are no standard techniques for solving non-linear differential equations. Lie had developed an approach to try to determine substitutions, whichcould be used to reduce the order of an ODE, or the number of independent variables of a PDE. This field has made dramatic advances under the name of "symmetry analysis". In this course Lie groups, local Lie groups and Lie algebras will be reviewed. Then the symmetries of algebraic and differential equations will be discussed. Next the techniques for finding the symmetries of an ODE, and their use for solving it will be presented. This will be extended tosystems of ODEs. The technique of finding differential invariants will be discussed with reference to some particular examples. The symmetries of PDEs will also be discussed and some examples presented.
Core Contents: Lie groups, local Lie groups and Lie algebras. Symmetries of algebraic and differential equations. Techniques for finding the symmetries of an ODE and their use for solving it. Extension to systems of ODEs. Differential invariants. The symmetries of PDEs. Techniques for finding the symmetries of a PDE, and their use for reducing the number of independent variables.
Detailed Curse Contents: One-parameter group of point transformations and their generators, Transformation laws, Extensions of transformations. Generators of point transformations and their prolongation; first formulation of symmetries; ODEs and PDEs of 1st order, Second formulation of symmetries Lie symmetries of 1st and 2nd order ODEs. Lie symmetries of 2nd order ODEs; higher order ODEs and linear nth order ODEs. The use of symmetries to solve 1st order ODEs. Lie algebras for infinitesimal generators. Examples of Lie Algebras. Subgroups and subalgabras; Invariants and Differential Invariants. The use of symmetries for solving 2nd order ODEs admitting a G_2. Second integration strategy. The use of symmetries for solving 2nd order ODEs admitting more than two symmetries. Higher order ODES admitting more than one Lie point symmetry. System of second order differential equations. Symmetries more general than Lie point symmetries. Symmetries of partial differential equations. Use of symmetries for solving partial differential equations of 1st order. 2nd order PDEs; Generating solutions by Symmetry transformations.
Course Outcomes: On successful completion of this course, students will be able to

- understand the basic concepts of the Lie point symmetries
- determine the symmetries of differential equations
- use symmetries to get the solutions or reduce order of ordinary differential equations
- determine the symmetries of system of ordinary differential equations
- determine the Noether symmetries of differential equations
- understand the need of contact symmetries of differential equations

Textbook: Hans Stephani, Differential Equations: Their Solution Using Symmetries, Cambridge University Press 1990

Reference book: N. H. Ibragimov, Elementary Lie Group Analysis and Ordinary Differential Equations, John Wiley and Sons1999.

Weekly Breakdown		
Week	***Section***	***Topics***
1	2.1-2.3	One-parameter group of point transformations and their generators, Transformation laws, Extensions of transformations.
2	2.4, 3.1-3.2	Generators of point transformations and their prolongation; first formulation of symmetries; ODEs and PDEs of 1st order
3	3.3-3.4, 4.1-4.2	Second formulation of symmetries Lie symmetries of 1st and 2nd order ODEs.
4	4.3, 4.4	Lie symmetries of 2nd order ODEs; higher order ODEs and linear nth order ODEs.
5	5.1-5.2	The use of symmetries to solve 1st order ODEs.
6	6.1-6.2	Lie algebras for infinitesimal generators. Examples of Lie Algebras.
7	6.3-6.5	Subgroups and subalgabras; Invariants and Differential Invariants.
8	7.1-7.2	The use of symmetries for solving 2nd order ODEs admitting a G2.
9	**Mid Semester Exam**	
10	7.3-7.4	Second integration strategy.
11	7.5, 8.1-8.3	The use of symmetries for solving 2nd order ODEs admitting more than two symmetries.
12	9.1-9.5	Higher order ODES admitting more than one Lie point symmetry.
13	10.1-10.3	System of second order differential equations.
14	11.1-11.5	Symmetries more general than Lie point symmetries.
15	15.1-15.3 16.1	Symmetries of partial differential equations. Use of symmetries for solving partial differential equations of 1st order.
16	16.2, 17.1-17.4	2nd order PDEs; Generating solutions by Symmetry transformations.
17		Review
18	**End Semester Exam**	

MATH-906 Symmetry Methods for Differential Equations-II

Credit Hours: 3-0
Prerequisites:MATH-905 Symmetry Methods for Differential Equations-I

Course Objectives: After having completed this course, the students would be expected to find and use symmetries of partial differential equations; they will be familiar with the Noether and the Lie-Backlund symmetries; they will know the Potential symmetries for differential equations and inherited symmetries.

Core Contents: Review of symmetry analysis for ODEs including multi-parameter groups, canonical variables, invariants, reduction of order, multi-parameter groups, integration by two symmetries, invariant solutions; contact and higher order symmetries, use of integrating factors, connection with symmetries; potential symmetries for ODEs and inherited symmetries. Review of symmetry analysis for PDEs; invariance for scalar and systems of PDEs, symmetries of DEs and its applications to boundary value problems; Noether's theorem, variational symmetries, conservation laws and higher order conservation laws, Euler-Lagrange and Lie-Backlund operators, Lie-Backlund symmetries, recursion operators for Lie-Backlund symmetries, mappings of infinitesimal generators from specified PDEs to target PDEs, invertible mappings for nonlinear systems of PDEs; potential symmetries for PDEs and inherited symmetries.

Course Outcomes: Students have good understanding of use of symmetries of differential equation. Students are able to solve boundary value problem. They have enough knowledge about the Potential symmetries for differential equations and inherited symmetries.

Text Book: George W. Bluman and S. Kumei, Symmetries and Differential Equations Springer Verlag 1989.

Reference Book:

1. Nail H. Ibragimove, Elementary Lie Group Analysis and Ordinary Differential Equations, John Wiely & Sons 1999.
2. George W. Bluman and Stephen C. Anco, Symmetry and Integration Methods for Differential Equations Springer Verlag 2002.

Weekly Breakdown		
Week	***Section***	***Topics***
1		Review of symmetry analysis for ODEs.
2	3.5-3.7	Applications to Boundary Value Problems
3	4.1 -4.2	Invariance for scalar of PDEs
4	4.3-4.4	Invariance for systems of PDEs, application to boundary value problems.
5	4.4	Formulation of invariance of a BVP for a PDE
6	4.4	Incomplete invariance for a linear system of PDEs
7	5.1, 5.2.1-2	Noether's theorem, variational symmetries and conservation laws; Boyer's Formulation
8	5.2.3-5.2.4	Equivalent Classes of Lie-Backlund transformation; Lie Bucklund Symmetries
9	**Mid Semester Exam**	
10	5.2.5-5.2.7	Finding variational symmetries; Noether's formulation; higher order conservation laws
11	5.3	Recursion operators for Lie-Backlund symmetries Mappings of infinitesimal generators from specified PDEs to target PDEs.
12-13	6.1 - 6.4	Introduction; Notations; Mappings of infinitesimal generators; Invertible mappings for nonlinear systems of PDEs to linear PDEs
14	6.5 - 6.6	Invertible mappings of linear PDEs to linear PDEs with constant coefficient.
15	7.1, 7.2	Potential symmetries for PDEs and inherited symmetries
16	7.3, 7.4	Potential symmetries for ODEs and inherited symmetries. Review of material.
17	-	Review
18	**End Semester Exam**	

MATH 908 Fixed Point Theory

Credits Hours: 3-0
Prerequisites: Some basic knowledge of Analysis

Course objectives: Aims: to teach elements of the metric fixed point theory with applications. Objectives: a successful student will:

- Be acquainted with some aspects of the metric fixed point theory;
- Have sufficient grounding in the subject to be able to read and understand some research texts;
- be acquainted with the principal theorems as treated and their proofs and able to use themin the investigation of examples.

Detailed Course Contents: The course includes Lipschitzian, contraction, contractive & non-expansive mappings, Banach's contraction principal with application to differential and integral equations, Brouwer's fixed point theorem with applications, Schauder's fixed point theorem with applications, uniformly convex and strictly convex spaces, properties of non- expansive mappings, Extension's of Banach's contraction principal, Fixed Point Theory in Hausdorff Locally Convex Linear Topological Spaces, Contractive and non-expansive Multivalued maps.

Text Book:
Introductory Functional Analysis with Applications, E. Kreyszig, John Wiley & Sons, New York, 1978.(IFAA)
Fixed Point Theory and Applications, Agarwal, R., Meehan, M., &O'Regan, (Cambridge Tracts in Mathematics). Cambridge: Cambridge University Press, 2001. (FPTA)

Reference Books:

1. An Introduction to Metric Spaces and Fixed Point Theory, M. A. Khamsi, W. A. Kirk, John Wiley & Sons, New York, 2001.
2. Fixed Point Theory, V. I. Istratescu, D. Reidel Publishing Company, Holland, 1981.

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1, 1.2, 1.3 (IFAA)	Metric Spaces, Examples of metric spaces. Open sets closed sets.
2	2.2, 2.3 (IFAA)	Normed spaces, Banach spaces, Properties of normed spaces
3	5.1 (IFAA)	Banach fixed point theorem
4	5.2 (IFAA)	Applications of Banach's Theorem to Linear equations
5	5.3 (IFAA)	Applications of Banach's Theorem to Differential equations
6	5.4 (IFAA)	Applications of Banach's Theorem to Integral equations
7	1 (FPTA)	Contractions
8	2(FPTA)	Non-expansive maps
9	**Mid Semester Exam**	
10	3(FPTA)	Continuation Methods for Contractive and non-expansive mappings
11	4(FPTA)	The Theorems of Brouwer , Shauder
12	5(FPTA)	Nonlinear alternatives of Leray-Shauder type
13	6(FPTA)	Continuation Principles for Condensing Maps
14	7(FPTA)	Fixed point Theorem in Conical Shells
15	8(FPTA)	Fixed Point Theory in Hausdorff Locally Convex Linear Topological Spaces
16	9(FPTA)	Contractive and non-expansive Multivalued maps
17		Review
18	**End Semester Exam**	

MATH 909 Continuum Mechanics-I

Credits Hours: 3-0
Prerequisites: None

Course Objectives: This lecture course aims to introduce students to the basic concepts of Continuum Mechanics and linear elasticity

Core Content: Tensors, basic constitutive laws of linear elasticity, stress and strain tensors in linear elasticity, elastic materials and symmetries, elasticity and problems related to reflection, refraction of waves, surface waves and wave guides.

Detailed Contents: Tensors: Definition of a tensor of order 2 and its extension to higher orders in a recursive manner. Change of basis. Covariant and contravariant tensors. Tensor algebra.
Symmetry in elastic materials: Periodicity in crystals, lattices, unit cell. The seven crystal systems.
Effect of symmetry on tensors: Reduction of the number of independent components of a tensor due to crystal symmetry, matrices for group symmetry elements in crystals, effect of a centre of symmetry and an axis of symmetry.
Static elasticity: The strain and stress tensors, equilibrium conditions. Hooke's Law. The elasticity tensor. Elastic energy in a deformed medium. Restrictions imposed by crystal symmetry on the number of independent elastic moduli.
Dynamic elasticity: Propagation equation, properties of elastic plane waves. Propagation along directions linked to symmetry. Elastic waves in an isotropic medium.
Reflection and refraction: Reflection of an SH wave from the surface of a half space. Reflection and refraction of a P-wave and and SV wave. Mode conversion.
Surface waves: The Rayleigh wave, uniqueness of the wave speed. The Love wave.
Wave guides: The Rayleigh Lamb dispersion relation for an isotropic plate. Lamb waves in an anisotropic plate.
Learning Outcomes: On successful completion of this course, students are expected to have:
- Understood mathematical definition of a tensor of rank n as a bilinear mapping from V^{n-1} to V, where V is a vector space. He/she should be adept at tensor algebra.
- Understood the symmetry groups associated with various classes of elastic materials.
- Understood equations of motion describing the dynamics of a continuum.
- Understood wave propagation in an anisotropic material.
- Understood the theory of Rayleigh waves, Love waves and Rayleigh-Lamb waves in a wave guide.
- Understood reflection and transmission of waves across an interface.

Text books:
ED: E. Dieulesaint and D. Royer, *Elastic Waves in Solids-I, Free and Guided Waves*, John Wiley and Sons.(2000)
JDA: J. D. Achenbach, *Wave Propagation in Elastic Solids*, North Holland.(1973)

Reference books:
1. N.D. Critescu, E.M. Cracium and E. Soos, Mechanics of Elastic Components, Chapman and Hall.
2. T.C.T. Ting, Anisotropic Elasticity, Oxford University Press.

Weekly Breakdown		
Week	***Section***	***Topics***
1	Instructor's choice for book	Vector space, tensor of rank 2 as a linear mapping from V to V. Orthonormal bases.
2	-do-	Tensor of rank n. Tensor algebra.
3	ED 2.1-2.2	Symmetry in elastic materials, seven crystal systems.
4	ED 2.6	Reduction of number of independent components of a tensor due to symmetry.
5	ED 3.1	The strain and stress tensors. Physical interpretation of components. Equilibrium conditions.
6	ED 3.2	The elasticity tensor
7	ED 3.2	Restrictions imposed by crystal symmetry on the number of independent elastic moduli. Matrix representations for the seven crystal systems.
8	JDA 1.2	Linearzed theory of wave propagation, Waves in one dimensional longitudinal stress,
9	**Mid Semester Exam**	
10	JDA 2.4, 2.10	Elastic waves in an isotropic medium. The scalar and vector potentials.
11	JDA 4.1, 4.2	Plane waves, Time-harmonic plane waves
12	JDA 4.4 5.1-5.2, 5.4	Two dimensional wave motion with axial symmetry Joined half spaces
13	JDA 5.5-5.7	Reflection of an SH wave from the free surface of a half space. Reflection and transmission of a P wave and an SV wave, mode conversion.
14	JDA 5.11	The Rayleigh wave. Uniqueness of the phase speed
15	JDA 6.6	Propagation in a layer. Love wave.
16	JDA 6.7-6.8	Wave guides. The Rayleigh-Lamb dispersion relation in an isotropic plate. Analysis of the shape of the spectrum. The anomalous Lamb modes.
17		Review
18	**End semester Exam**	

MATH-910 Continuum Mechanics-II

Credit Hours: 3-0
Prerequisites: MATH-909 Continuum Mechanics-I

Course Objectives: The course objectives are to give a basic knowledge of three dimensional continuum mechanics, its challenge and main concepts.

Core Contents: Configurations, analysis of motion, analysis of stress, elastic materials, boundary value problems, incremental deformations, applications of the theory to specific problems in elasticity

Detailed Course Contents: Tensor Theory Review: Cartesian Tensors, Tensor Algebra, Tensor Fields, Analysis of deformation and Motion: Kinematics, Analysis of deformation andStrain, Analysis of Motion, Objectivity of tensor fields, Balance Laws, Stress and Field Equations: Mass Conservation, Momentum balance equations. The Cauchy stress tensor, The nominal stress tensor
Elasticity: Constitutive laws for simple materials, Cauchy Elastic Materials, Green Elastic Material, Application to simple homogenous deformations, Boundary Valve Problems: Problems for unconstrained materials and constrained materials, Incremental Elastic Deformations: Incremental constitutive relations, Introduction to strong Ellipticity condition, Applications of incremental elasticity

Course Outcomes:
At the end of the course, the student should be able to:

- Describe the general model of nonlinear elastic deformation in the reference (Lagrangian) and current (Eulerian) system of references.
- Formulate the basic boundary value problem of finite elasticity and solve the governing equations for a selection of problems for (internally) constrained and unconstrained isotropic materials.
- Connect the linear theory of elasticity with the non-linear theory
- Use the theory of superposition of incremental elastic deformations on finite deformations
- Solve analytically or numerically specific problems, including the Rivlin cube, torsion of a bar, extension and inflation of a cylindrical tube, inflation of a spherical shell, buckling of a rod etc.

Text Book: Raymond W Ogden, Non-linear elastic deformations, Dover Publications 1997 and Ellis Horwood 1984

Recommended Book: Romesh C. Batra, Elements of Continuum Mechanic, AIAA Education Series (2006)

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.2, 1.3, 1.5	Tensor Theory Review: Cartesian Tensors, Tensor Algebra, Tensor Fields
2	2.1, 2.2	Analysis of deformation and Motion Kinematics, Analysis of deformation and Strain
3	2.2, 2.3	Continued: Analysis of deformation and Motion, Analysis of Motion
4	2.4 3.1 – 3.2	Objectivity of tensor fields Balance Laws, Stress and Field Equations Mass Conservation
5	3.2 - 3.3	Momentum balance equations, The Cauchy stress tensor
6	3.3	The Cauchy stress tensor (Cont)
7	3.4-3.5	Cont. Balance Laws, Stress and Field Equations The nominal stress tensor, Definition of Nominal Stress, The Lagrange Field Equations
8	4.1-4.2	Elasticity Constitutive laws for simple materials, Cauchy Elastic Materials
9	**Mid Semester Exam**	
10	4.2 - 4.3	Cont. Elasticity Cauchy Elastic Material (Cont.), Green Elastic Material
11	4.3 - 4.4	Cont. Elasticity Green Elastic Material, Application is simple homogenous deformations. (Optional)
12	5.1- 5.2	Boundary Valve Problems Formulation of BVP, Problems for unconstrained materials.
13	5.3	Cont. Boundary Valve Problems: Problems for materials with internal constrains
14	6.1	Incremental Elastic Deformations Incremental constitutive relations
15	6.1	Cont. Incremental constitutive relations.
16	6.2.7 , 6.3	Introduction to strong Ellipticity condition, Applications of incremental elasticity
17	-	Review
18	**End semester Exam**	

MATH-911 Special Functions

Credit Hours: 3-0
Prerequisite: None

Objectives and Goals: This course deals with the theory of functions of real and complex variables. While the original definition of a function may be in a more limited domain it can often be extended to larger domains by analytic continuation. As such, integral transforms that extend the domain of applicability are needed to study the functions in themselves. We will first discuss integral transforms and "fractional calculus" and go on to the special functions used in other areas of mathematics, in Statistics and in number. We then go on to the special functions of mathematical physics that originated as solutions of 2^{nd} order linear ordinary differential equations and their continuation by integral representations.

Core Contents: Transform Methods, Fractional Calculus, Special Functions.

Detailed Course Contents: The integral operator and integral transforms. Linear and non- linear integral transforms. Fourier transforms of classical functions and conditions for existence. Properties of Fourier transform. Convolutions properties of Fourier transform. Distributions and generalized functions. Fourier transforms of generalized functions. Poisson summation formulae and applications. The Laplace transform and conditions for its existence. Basic properties of Laplace transform. Convolutions. Inverse Laplace transforms. Differentiation and integration of Laplace transforms. Use of Laplace transforms for differential and integral equations. Fractional calculus and its applications. Fractional differential and integral equations. The Hilbert transform and its properties. Extension to the complex domain. The Steiltjes transform its properties and inversion theorems. The Mellin transform. The gamma and beta functions and their integral representations. Properties and asymptotic expansion of the gamma function. The probability integral and its properties for real and complex domains. The exponential and logarithmic integrals. Hypergeometric functions and Legendre functions. The hypergeometric series and its analytic continuation. Properties of the hypergeometric functions. Confluent hypergeometric functions. Generalized hypergeometric functions.

Learning Outcomes: On successful completion of this course, students will be able to:

- Understand the concepts of integral transforms.
- Understand the notion of fractional calculus.
- Know the transform methods and special functions with their properties and applications.

Text books:

L. Debnath and D. Bhatta, Integral Transforms and Their Applications Chapman & Hall/CRC; Second Edition (October 2006)

N.N. Lebedev, Special Functions and their applications (tr. R.R. Silverman) Dover Publications (Revised Editions, June 1972)

Reference Books:

1. M. Ya. Antimirov, A. A. Kolyshkin and Remi Vaillancourt, Applied Integral Transforms, The American Math. Society, (1993)
2. Nikiforov and Uvarov, Special Functions of Mathematical Physics, Springer, 1988

Weekly Breakdown		
Week	***Section***	***Topics***
1	2.1-2.5, 2.9	Fourier transforms of classical functions and conditions for existence. Properties of Fourier transform. Convolutions properties of Fourier transform.
2	2.10-2.13, 3.1-3.4	Fourier transforms of generalized functions. Poisson summation formulae and applications to the solution of differential and integral equations. The Laplace transform and conditions for its existence. Basic properties of the Laplace transform.
3	3.4 – 3.7	Convolutions, Inverse Laplace transforms. Differentiation and integration of Laplace transforms.
4	5.1 – 8.4	Fractional calculus and its applications. Fractional differential and integral equations
5	6.1-6.3	Laplace transform of fractional integrals and derivatives, Mittage-Leflller function and its properties, Fractional ordinary differential equations.
6	6.4,6.5	Fractional integral equations, Initial value problems for fractional differential equations
7	8.1-8.4	Mellin Transforms: Properties and application of Mellin transforms
8	8.5-8.7	Mellin transform of fractional integrals and derivatives
9	**Mid Semester Exam**	
10	9.1-9.4	The Hilbert transform and its properties, Extension to the complex domain
11	9.7-9.8	The Steiltjes transform its properties and inversion theorems.
12	NNL 1.1 – 1.5	The gamma and beta functions and their integral representations. Properties and asymptotic expansion of the gamma function. Incomplete gamma function.
13	2.1 - 2.4	The probability integral and its properties for real and complex domains. Asymptotic representation of probability integrals.
14	3.1 - 3.4	The exponential and logarithmic integrals. Asymptotic representation of exponential integrals.
15	7.1 – 7.6	Hypergeometric functions and Legendre functions. The hypergeometric series and its analytic continuation
16	9.1 – 9.5 9.7, 9.8	Properties of the hypergeometric functions. Confluent hypergeometric functions. Generalized hypergeometric functions
17		Review
18	**End semester exam**	

MATH-941 Graph Theory

Credit Hours: 3-0
Prerequisites: None

Course Objectives: Graph theory is a stand-alone branch of pure mathematics that has links across the mathematical spectrum. The primary objective of the course is to introduce students to the beautiful and elegant theory of graphs, focusing primarily on finite graphs.
Previous Knowledge: Basic knowledge of linear algebra is needed.

Core Contents: Basics of graph theory, Path, Cycles, Trees, Matchings, Connectivity and Network Flows, Coloring, Planar graphs.

Detailed Contents: The basics of graph theory: Definition of a graph, graphs as models, matrices, isomorphism, decomposition, paths, cycles, trails, bipartite graphs, Eulerian circuits, vertex degrees and counting, directed graphs.
Trees: Properties of trees, distances in trees and graphs, spanning trees in graphs, decomposition and graceful labeling, minimum spanning trees, shortest paths, trees in computer science.
Matching: Maximum matchings, Hall's matching condition.
Connectivity: Connectivity, edge connectivity, blocks, 2-connected graphs, maximum network flow.
Coloring: Vertex coloring, chromatic number, clique number, upper bounds on chromatic number.
Planar graphs: Drawing in the plane, dual graphs, Euler's Formula.

Text Book: Douglas B. West, Introduction to Graph Theory, Second Edition, Pearson Education Inc, 2001.

Reference Books:

1. Reinhard Diestel, Graph Theory, Third edition, Springer 2005.
2. J.A. Bondy and U.S.R. Murty, Graph Theory, Springer 2010.
3. B. Bollobas, Modern Graph Theory, Springer 1998.

Weekly Breakdown	
Week	**Topics**
1	Definition of graphs: loops, multiple edges, simple graphs, neighbors. Graph as models: Complement, clique, independent set, bipartite graphs
2	Chromatic number, k-partite graphs, path, cycle, subgraphs. Matrices and Isomorphism: adjacency matrix, incidence matrix, degree of vertex
3	Isomorphism, n-cycle, complete graph, complete bipartite graphs. Decomposition: self-complementary graphs, decomposition
4	Triangle, paw, claw, kite, Petersen graph, girth. Connection in graphs: walks, trail, u,v-walk and path, internal vertices, length of walk and path.
5	Connected and disconnected graphs, components of graph, isolated vertex, cut-edge, cut-vertex, induced subgraphs, union of graphs, Eulerian graphs, Eulerian circuits, even graph
6	Vertex degrees and counting: degree of vertex, regular and k-regular graphs, neighborhood, order of a graph, Counting and bijections: degree sum formula, k-dimensional cube. Graphic sequence, introduction of directed graphs
7	Trees: acyclic graph, forest, leaf, spanning subgraphs, spanning trees, star, properties of trees
8	Distances in trees and graphs: distance, diameter, eccentricity, radius, center of a graph, Wiener index, contraction of edges, graceful labelling
9	**Mid Semester Exam**
10	Minimum spanning tree: Kruskal Algorithm, Shortest path: Dijkstra's Algorithm
11	Trees in Computer Science: Rooted tree, children, ancestors, descendants, rooted plane tree, binary tree, left child, right child
12	Matchings: matching, perfect matchings, maximum and maximal matchings, M-alternating and augmenting paths, symmetric difference, Hall's matching condition
13	Connectivity: vertex cut, connectivity and k-connected graphs, edge-connectivity, edge-connectivity and k-edge-connected graphs,
14	Network Flow Problems: Network, capacity, source and sink vertex, flow, maximum network flow, Ford-Fulkerson labeling algorithm
15	Coloring of graphs: k-coloring, proper coloring, k-colorable graphs, chromatic number, k-chromatic graphs, greedy coloring algorithm
16	Planar graphs: curve, polygonal curve, crossing, planar graphs, planner embedding, closed curve, simple curve, region, faces, dual graphs, Euler's formula
17	Review
18	**End Semester Exam**

MATH-943 Convex Analysis

Credit hours: 3-0
Prerequisites: ~~MATH 802 Analysis~~

Course Objectives: Although the systematic study of convex sets started by the end of the 19th century, convexity only became an independent branch of mathematics by the middle of the 20th century. Convexity combines conceptual tools from geometry, analysis, linear algebra and topology, and plays a crucial role in number theory, optimization, inequality theory, combinatorial geometry and game theory. The course is focused on convex sets and convex functions, showing applications to optimality theory in convex programming and conjugacy theory.

Core Contents: Basic concepts of convex analysis, Topological properties of convex functions, Duality correspondence, Representation and inequalities and Bifunctions and generalized convex program.

Detailed Course Contents: Affine sets, convex sets and cones, the Algebra of convex sets, convex functions, functional operations, relative interiors of convex sets, closures of convex functions, some closeness criteria, continuity of convex functions, separation theorems, conjugates of convex functions, support functions, polers of convex sets and functions, dual operations, Caratheodory's theorem, extreme points and faces of convex sets, polyhedral convex sets and functions, some applications of polyhedral convexity, Helly's theorem and systems of inequalities, directional derivatives and sub gradients, constrained extremumproblems, saddle functions and minimax theory.

Learning Outcomes: Students are expected to understand the fundamentals of convex analysis, Topological properties of convex functions, Duality correspondence, Representation and inequalities.

Text Book: R. Tyrrel Rockafeller, Convex Analysis, Princeton University press, 1970.

Weekly Breakdown		
Week	***Section***	***Topics***
1	Part I Sec. 1,2	Affine sets, convex sets and cones
2	Part I Sec. 3,4	The Algebra of convex sets, convex functions
3	Part II Sec. 5, 6	Functional Operations, Relative interiors of convex sets
4	Part II Sec. 7, 8	Closures of convex functions, Recession cones and unboundedness
5	Part II Sec. 9, 10	Some closeness criteria, Continuity of convex functions
6	Part II Sec. 11, 12	Separation Theorems, Conjugates of convex functions
7	Part III Sec. 13, 14	Support function
8	Part III Sec. 14, 15	Polars of Convex sets, polars of convex functions
9	**Mid Semester Exam**	
10	Part III Sec. 16	Dual operations
11	Part IV Sec. 17, 18	Caratheodorey's Theorem, Extreme points and faces of convex sets
12	Part IV Sec. 19, 20	Polyhedral Convex sets and functions, Some applications of Polyhedral convexity
13	Part IV Sec. 21, 22	Helly's Theorem and systems of inequalities, Linear inequalities
14	Part V Sec. 23,24	Directional derivatives and sub gradients, Differential continuity and Monotonicity
15	Part VI Sec 27, 28	The minimum of a convex function, Ordinary convex programs and Lagrange multipliers
16	Part VI Sec 29, 30	Bifunctions and generalized convex program, Fenchel's duality theorem
17		Review
18	**End Semester Exam**	

MATH-944 Semigroup Theory of Operators

Credit Hours: 3-0

Prerequisite: None

Course Objectives: PhD/M.Phil and graduate students of functional analysis, applied mathematics, physics and engineering will find this an invaluable introduction to the subject. Main aim is to introduce students to the solutions of problems involving evolution equations via the theory of semigroup of operators. This course will enable the students to proceed to advanced textbooks and to many research papers devoted to the use of semigroups in the study of evolution systems.

Core Contents: Spectral Theory, Cauchy's Functional Equation, Semigroups on Banach and Hilbert spaces, Strongly continuous semigroups, Well-posedness for evolution equations, Semilinear problems.

Course Contents: Spaces and operators, spectral theory, fixed point theorem, uniformly continuous operator semigroups, semigroups on Banach spaces, semigroups on Hilbert spaces, strongly continuous semigroups, generators of semigroups, Hille-Yosida theorems, dissipative and m-dissipative operators, construction of semigroups, perturbation of generators, abstract Cauchy problems, inhomogeneous Cauchy problems, semilinear ACP, mild solutions, strong solutions.

Learning Outcomes: Students are expected to understand Spectral Theory, Cauchy's Functional Equation, Semigroups on Banach and Hilbert spaces, Strongly continuous semigroups, and applications of semigroup operator theory in differential equations and functional equations.

Text Books:

A. Bellani-Morante and A. C. McBride, Applied Nonlinear Semigroups, John Wiley & Sons (Referred asBM)

K-J Engel and R. Nagel, One Parameter Semigroups for Linear Evolution Equations Springer (Referred as EN)

Weekly Breakdown		
Week	***Section***	***Topics***
1	BM 1.7-1.11	Spaces and Operators, spectral Theory, Fixed Point Theorem
2	EN Chapter 1 1.1-1.4, 2.1-2.11	Cauchy's Functional Equation, Finite Dimensional Systems
3	Chapter 1 Section 3	Uniformly continuous operator semigroups, semigroups on Banach spaces, Semigroups on Hilbert spaces
4	Chapter 1 4.1 – 4.8	Multiplication Semigroups, Translation semigroups
5	Chapter 1 Section 5	Strongly continuous semigroups and its basic properties
6	Chapter 2 1.1-1.7	Construction and examples of strongly continuous semigroups
7	1.8-1.14	Generator of Semigroups and their resolvents
8	Chapter 2 2.1-2.11	Standard construction of similar semigroups, rescaled semigroups, subspace semigroups, quotient semigroups, adjoint semigroups, Product semigroups
9	**Mid Semester Exam**	
10	3.1-3.11	Hille-Yosida Generation Theorems
11	3.13-3.23	Dissipative Operators and Contractive Semigroups
12	4.1-4.15	Special classes of semigroups
13	Chapter 2 6.1-6.11 BM 2.5	Well-posedness for evolution equations, abstract Cauchy problems, Inhomogeneous abstract Cauchy problem and its strong solutions
14	Chapter 3 1.-1.15 BM 2.4	Perturbation of Generators, the Trotter-Kato theorems
15	BM 3.1-3.2	Semilinear problems
16	BM 3.3-3.4	strong solutions, mild solutions
17	Review	
18	**End Semester Exam**	

MATH-945 Lie Group Representations

Credit Hours: 3-0
Prerequisite: None

Objectives and Goals: The representation theory of Lie groups plays an important role in the mathematical analysis of the elements. In particular, the study of representations of the Lie group SO(3) leads to an explanation of the Periodic Table of the chemical elements, the study of representations of the Lie group SU(2) naturally leads to the famous Dirac equation describing the electron.
The objective of the course is to introduce the concepts of matrix Lie groups and exponentials, Lie algebras and basic representation theory. After completion of the course students are expected to be equipped with the concepts of representation theory of Lie groups and are able to apply the tools learnt in different areas like general relativity, string theory etc.
Core Contents: Matrix Lie Groups, The Matrix Exponential, Lie Algebras, Basic Representation Theory.
Course Contents: Matrix Lie Groups: Definitions, Examples, Topological Properties, Homomorphisms, Lie Groups.
The Matrix Exponential: The Exponential of a Matrix, Computing the Exponential, The Matrix Logarithm, Further Properties of the Exponential, The Polar Decomposition.
Lie Algebras: Definitions and First Examples, Simple, Solvable, and Nilpotent Lie Algebras, The Lie Algebra of a Matrix Lie Group, Examples, Lie Group and Lie Algebra Homomorphisms, The Complexification of a Real Lie Algebra, The Exponential Map, Consequences of Theorem 3.42.
Basic Representation Theory: Representations, Examples of Representations, New Representations from Old, Complete Reducibility, Schur's Lemma, Representations of sl(2;C), Group Versus Lie Algebra Representations, A Nonmatrix Lie Group.
Course Outcomes: Students are expected to understand Matrix Lie Groups, The Matrix Exponential, Lie Algebras and Basic Representation Theory.

Learning Outcomes: Students are expected to understand Matrix Lie Groups, The Matrix Exponential, Lie Algebras, Basic Representation Theory.

Text Book: Brian C. Hall, Lie Groups, Lie Algebras, and Representations (2nd Ed.), Springer International Publishing (2015).

Reference Books:

1. Andrew Baker, Matrix Groups: An Introduction to Lie Group Theory,Springer (2002).
2. MariánFecko, Differential Geometry and Lie Groups for Physicists, Cambridge University Press (2006).
3. Robert Gilmore, Lie Groups, Lie Algebras and Some of Their Applications, Dover Publications (2006).

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1,1.2	Matrix Lie Groups: Definitions, Examples.
2	1.3	Topological Properties.
3	1.4,1.5,2.1	Homomorphisms, Lie Groups. The Matrix Exponential: The Exponential of a Matrix.
4	2.2 - 2.4	Computing the Exponential, The Matrix Logarithm, Further Properties of the Exponential.
5	2.5, 3.1	The Polar Decomposition. Lie Algebras: Definitions and First Examples.
6	3.2,3.3	Simple, Solvable, and Nilpotent Lie Algebras, The Lie Algebra of a Matrix Lie Group.
7	3.4, 3.5	Examples, Lie Group and Lie Algebra Homomorphisms.
8	3.6, 3.7	The Complexification of a Real Lie Algebra, The Exponential Map.
9	**Mid Semester Exam**	
10	3.8	Consequences of Theorem 3.42.
11	4.1,4.2	Basic Representation Theory: Representations, Examples of Representations.
12	4.3	New Representations from Old.
13	4.4,4.5	Complete Reducibility, Schur's Lemma.
14	4.6	Representations of sl(2;C).
15	4.7	Group Versus Lie Algebra Representations.
16	4.8	A Nonmatrix Lie Group
17	-	Review
18	**End Semester Exam**	

MATH-946 Category Theory

Credit Hours: 3-0
Prerequisite: Fundamental knowledge of Topology & Algebra

Objectives and Goals: This course aims at introducing students to the concepts of categories, functors and natural transformations. On successful completion of this course, students will know categories, discrete objects, indiscrete objects, functors, properties of functors, natural transformations, products, co-products, equalizers, co-equalizers, pullbacks, pushouts, limits and co-limits.

Course Contents: Categories, morphisms, concrete categories, abstract categories, sections, retractions, isomorphism, monomorphisms, epimorphisms, initial objects, final objects and zero objects, functors, hom- functors, Properties of functors, natural transformations and natural isomorphisms, equalizer and coequalizer, products and coproducts, discrete and indiscrete objects, sources and sinks, pullbacks, pushouts, limit, co-limits.

Course Outcomes: Students are expected to understand:

- Categories, morphisms, abstract and concrete categories
- Sections, Rectractions, Isomorphism, Mono and Epimorphism
- Initial, Final and Zero Objects
- Functors and Properties of Functors
- Natural transformations and Natural isomorphism
- Equalizer, Coequalizer, Product and Coproduct
- Discrete and Indiscrete objects
- Sources and Sinks
- Pullbacks and Pushouts
- Limits and Colimits

Text Books:
S. Awodey, "Category Theory", Oxford University Press (2nd edition), 2010.
J. Adamek, H. Herrlich, and G. E. Strecker, "Abstract and Concrete Categories, The Joy of Cats", Dover Publications, 2009.

Reference Books:
1. G. Preuss, "Foundations of Topology", Kluwer Academik Publisher, 2002.
2. S. Mac Lane, "Categories for working mathematicians", Springer, 2nd Edition, 1997.
3. D. I. Spivak, "Category theory for the Sciences", MIT press, 2013
4. M. Barr and C. Wells, "Category theory for Computing Science", Prentice hall international UK, 1990

Weekly Breakdown	
Week	***Topics***
1	Sets, Classes and conglomerates categories, Morphisms
2	Concrete Categories, Abstract Categories
3	Section, Retractions, Monomorphisms
4	Epimorphisms and Isomorphisms
5	Functors, Hom-functors
6	Properties of functors
7	Initial objects, Final objects and Zero objects
8	Fixed morphisms, Zero morphisms and Point categories
9	**Mid Semester Exam**
10	Natural transformation, Natural isomorphisms
11	Discrete and Indiscrete objects
12	Equalizer, Coequalizer
13	Products and Coproducts
14	Pullbacks, Pushouts
15	Sources and Sinks
16	Limit, Co-limits,
17	Review
18	**End Semester Exam**

MATH-949 Combinatorics

Credit Hours: 3-0
Prerequisites: None

Course Objectives: This course is for the students in MS/ PhD Mathematics program. The main objective of this course is to understand countable discrete structures.The main educational objectives of this course are:
To introduce the discrete structures and discrete mathematical models
To model, analyze, and to solve combinatorial and discrete mathematical problems.
It is also aimed to develop the ability in students to apply these techniques for solving the practical problems in optimization, computer science and engineering as well as to apply combinatorial techniques in other disciplines of mathematics like algebra, graph theory and geometry etc.

Core Contents: Classical Techniques, Generating functions, Recurrence relation, Combinatorial Numbers, Partition of Integers, Inclusion-Exclusion Principal and applications, Polya's enumeration theory, Chromatic Polynomials of graphs

Detailed Course Contents: Classical Techniques: Two Basic counting principals, Binomial, Multinomial numbers and multinomial formula, combinations with or without repetitions, Permutations and permutation with forbidden positions; Brief Introduction to graphs/discrete structures. Generating Functions: Generating Function Models, Calculating Coefficients of Generating Functions, Exponential Generation Functions. Partition of Integers: Partitions of integers (their properties, recurrence relations, generating functions). Recurrence Relation: Recurrence Relation Models, Divide-and-Conquer Relations, Solution of Linear Recurrence Relations, Solution of Inhomogeneous Recurrence Relations, Solution with Generating Functions. Inclusion-Exclusion Principals: Counting with Venn diagrams, Inclusion formula and its forms, Applications of Inclusion-Exclusion. Combinatorial Numbers: Stirling, Bell, Fibonacci and Catalan numbers (their recurrence relations, generating functions and applications to enumeration problems in graph theory and geometry). Polya enumeration theory: Equivalence and symmetry groups, Burnside's Theorem. Chromatic Polynomials: Fundamental Reduction Theorem, Chromatic Equivalence, Chromatic Uniqueness

Course Outcomes: This course is specially designed for students who want to choose pure mathematics as their specialty in general and more specifically who want to opt discrete mathematics as their research area. On successful completion of this course, students will be able
To understand the fundamental structures and techniques of combinatorial mathematics and importance of combinatorial techniques in comparison with other techniques
To explore the logical structure of mathematical problems,
To develop problem solving skills in combinatorial related problems and their applications.

Text Book: Alan Tucker, Applied Combinatorics (4th Edition, 2002) JohnWiley and Sons.

Reference Books:

1. John M. Harris, Jeffry L. Hirst, Micheal J. Mossinghoff, Combinatorics and Graph Theory, Springer, 2nd Edition, 2008.
2. V. Krishnamurthy, Combinatorics, theory and applications, Ellis Horwood Publ., Chichester,1986.

3. R . A. Brualdi, Introductory Combinatorics (5th Edition), 2010, Prentice Hall

Weekly Breakdown		
Week	***Section***	***Topics***
1	5.1, 5.2, 5.5	Two Basic Counting Principles, Simple Arrangements and Selections, Binomial Coefficients and Binomial formula, Multinomial formula
2	5.3, 5.4, 5.5	Arrangements and Selections with Repetitions, Multinomial Coefficients and multinomial formula, The Pigeonhole Principle, Distributions, Binomials Identities
3	6.1, 6.2, 6.3	Generating Functions Models, Calculating Coefficients of Generating functions and applications, Exponential Generating Functions
4	7.1, 7.2	Recurrence Relation Models, Divide-and-Conquer relations
5	7.3,7.4	Solutions of Linear Recurrence Relations, Solution of Inhomogeneous Recurrence Relations
6	7.5	Solutions with Generating Functions
7	8.2	Counting with Venn Diagrams, Inclusion-Exclusion principle and applications
8	8.3	Permutations with forbidden positions
9	**Mid Semester Exam**	
10	2.6.4, 2.6.6	Stirling numbers (First kind and second kind) and Bell numbers (their recurrence relations, generating functions), applications of these numbers to enumeration problems in graph theory and geometry
11-12	2.8.2, 2.8.3 9.1, 9.2	Fibonacci and Catalan numbers (recurrence relations, generating functions) and applications
13	9.3	Equivalence and symmetry groups, Burnside's Theorem,
14	6.3	Partitions of integers (their properties, recurrence relations, generating functions)
15	9.4	Polya's Theorem and applications
16	J. M. Harris 1.6.4	Chromatic polynomials in graph colorings (properties and the fundamental reduction theorem), Chromatic Equivalence and chromatic Uniqueness
17	-	Review
18	**End semester Exam**	

MATH-951 Mathematical Modelling-II

Credit Hour: 3-0
Prerequisites: MATH-822 Mathematical Modelling-I

Course objectives: This course introduces powerful mathematical modeling techniques with reference to specific problems in physics, engineering, ecology, biology, sociology and economics, using dimensional analysis. It requires some background in differential equations, linear algebra and a little matrix theory. The purpose is that students taking this course should be able to construct models and use them to obtain results for the problems modeled.

Core Contents: The course will review linear algebra, matrix algebra and systems of differential equations. It will cover transformation of units and the structure of physical variables, dimensional analysis, dimensional similarities and models law; the nature of mathematical modeling; qualitative behavior of both linear and nonlinear system, stability analysis and bifurcation of dynamical systems; terminology and solution of several differential equations models, general equilibrium solutions of some realistic models; chaos in deterministic continuous systems.

Detailed Course Contents: Mathematical Preliminaries, Matrices and Determinants, Rank of a Matrix, System of linear equations, Review of dimensional systems, transformation of dimensions, arithmetic of dimensions, structure of physical variables, number of sets of dimensionless product of variables.
Sequence of Variables in Dimension Set: Dimensional physical variables is present, physical variable of identical dimensions are present, independent and dependent variables.
Dimensional Modeling: Introductory remarks, homology, specific similarities.
Dimensional Modeling: Dimensional similarities, models law, Categories and relations, scale effect.
Linear Equation and Models: Some linear models, linear equations and their solution, homogenous and non-homogenous equations and their applications, dynamics of linear equation, some empirical models.
Nonlinear Equations and Models: Some nonlinear models, autonomous equations and their dynamics, Cobwebbing, derivatives and dynamics, some mathematical applications, bifurcation and period-doubling.
Modeling Change One Step at a Time: Introduction, compound interest and mortgage payments, some examples, compounding continuously.
Differential Equation Models: Carbon dating, age of the universe, HIV modeling.
Modeling in Physical Science: Introduction, calculus, Newton, and Leibniz, Rewriting Kepler's laws mathematically, generalization.
Learning Outcomes: Students are expected to understand:
Fundamentals of mathematical modeling.
Linear equation and models based on linear equations.
Non-linear equation and models based on non-linear equations.
Modeling change one step at a time.
Text Books:
F. R. Marotto (FRM), Introduction to Mathematical Modeling Thomson Brooks, 2006.
K.K. Tung (PET), Topics in Mathematical Modeling, Princeton University Press, 2007.
Thomas Szirtes (TS), Applied Dimensional Analysis and Modeling (Second Edition), Elsevier Inc., 2007.

Weekly Breakdown		
Week	***Chapt.***	***Topics***
1	TS 1	Mathematical Preliminaries, Matrices and Determinants, Rank of a Matrix, System of linear equations.
2	3,4, 5,7,10	Review of dimensional systems, transformation of dimensions, arithmetic of dimensions, structure of physical variables, number of sets of dimensionless product of variables.
3	14	Sequence of Variables in Dimension Set: Dimensional physical variables is present, physical variable of identical dimensions are present, independent and dependent variables
4-5	17	Dimensional Modeling: Introductory remarks, homology, specific similarities.
6	17	Dimensional Modeling: Dimensional similarities, models law, Categories and relations, scale effect.
7	FRM 2	Linear Equation and Models: Some linear models, linear equations and their solution, homogenous and non-homogenous equations and their applications, dynamics of linear equation, some empirical models.
8	3	Nonlinear Equations and Models: Some nonlinear models, autonomous equations and their dynamics, Cobwebbing, derivatives and dynamics, some mathematical applications, bifurcation and period-doubling.
9	**Mid Semester Exam**	
10	KKT 3, 4	Modeling Change One Step at a Time: Introduction, compound interest and mortgage payments, some examples, compounding continuously. Differential Equation Models: Carbon dating, age of the universe, HIV modeling.
11	5	Modeling in Physical Science: Introduction, calculus, Newton, and Leibniz, Rewriting Kepler's laws mathematically, generalization.
12	6	Nonlinear Population Models: An introduction to qualitative analysis using phase planes, population models, harvesting models, economic considerations, depensation growth models
13	7, 8	Discrete Time Logistic Map, Periodic and Chaotic Solutions: Logistic growth for non-overlapping generations, discrete map, sensitivity to initial conditions.Snowball Earth and Global Warming: Introduction, simple climate models, the equilibrium solutions
14	8	Snowball Earth and Global Warming: Stability, the global warming controversy, a simple equation for climate perturbation, solution of equilibrium global warming
15	10	Marriage and Divorce: Mathematical models of self-interaction and martial interaction, an example of validating couple, terminology, general equilibrium solutions.
16	11	Chaos in Deterministic Continuous System: Introduction, some history of Henri and Lorenz, the Lorenz equations as model of convection, chaotic waterwheel.
17		Review
18	**End Semester Exam**	

MATH-955 General Relativity and Cosmology

Credit Hours: 3-0
Prerequisite: None
Course Objectives: General Relativity (GR) is a physical theory of gravitation invented by Albert Einstein in the early twentieth century. The theory has strong mathematical setup, has immense predictive power, and has successfully qualified several experimental/observational experiments of astrophysics and cosmology. Black holes and relativistic cosmology are two main applications of GR. It is intended that GR and its major applications and achievements be discussed in the manner they deserve.
Core Contents: Special relativity revisited, Electromagnetism, The gravitational field equations, The Schwarzschild geometry, Schwarzschild black holes, Kerr metric, Further spherically symmetric geometries.
Detailed Course Contents: Special relativity revisited: Minkowskispacetime in Cartesian coordinates, Lorentz transformations, Cartesian basis vectors, Four-vectors and the lightcone, Four-vectors and Lorentz transformations, Four-velocity, Four-momentum of a massive particle, Four-momentum of a photon, The Doppler effect and relativistic aberration, Relativistic mechanics, Free particles, Relativistic collisions and Compton scattering, Accelerating observers, Minkowskispacetime in arbitrary coordinates.
Electromagnetism: The electromagnetic force on a moving charge, The 4-current density, The electromagnetic field equations, Electromagnetism in the Lorenz gauge, Electric and magnetic fields in inertial frames, Electromagnetism in arbitrary coordinates, Equation of motion for a charged particle, Electromagnetism in a curved spacetime.
The gravitational field equations: The energy–momentum tensor, The energy–momentum tensor of a perfect fluid, Conservation of energy and momentum for a perfect fluid, The Einstein equations, The Einstein equations in empty space, The weak-field limit of the Einstein equations, The cosmological-constant term.
The Schwarzschild geometry: General static isotropic metric, Schwarzschild solution, Brinkhoff's theorem, Gravitational redshift, geodesics in Schwarzschild geometry, radial trajectories of massive particles, Circular motion of massive particles, stability of massive particle orbits, trajectories of photons, circular motion of photons, stability of photon orbits, Experimental tests of general relativity: Precession of planetary orbits, The bending of light, Accretion discs around compact objects.
Schwarzschild black holes: singularities in Schwarzschild metric, radial photon worldlines, radial particle worldliness in Schwarzschild coordinates, Eddington Finkelstein coordinates, black hole formation, Spherically symmetric collapse of dust, tidal forces near a black hole, Kruskal coordinates, wormholes and Einstein Rosen bridge, The Hawking effect of black hole evaporation.
Further spherically symmetric geometries: Spherically symmetric geometries: metric for stellar interior, relativistic equations of stellar structure, Schwarzschild interior solution, metric outside a spherically symmetric charged mass, Riessner-Nordstrom geometry and solution, Radial photon trajectories in RN geometry, radial massive particle trajectories.
Kerr metric: The Kerr metric, Limits of the Kerr metric, Ker Neumann Metric (handouts). The Friedmann–Robertson–Walker geometry: The cosmological principle, synchronous commoving coordinates, homogeneity and isotropy of the universe, maximally symmetric 3- space, Friedmann-Robertson-Walker metric, geometrical properties of FRW metric, The cosmological redshift, The Hubble and deceleration parameters, Components of the cosmological fluid, Cosmological parameters, The cosmological field equations, General dynamical behaviour of the universe, Evolution of the scale factor, Analytical cosmologicalmodels.

Learning Outcomes: Students will understand of the theory and predictions of Einstein's general relativity. Students will be capable to read research papers and initiate research in general relativity. Students will be able to understand the dynamical evolution of the universeby studying cosmology.

Text Book: M.P. Hobson, G.P. Efstathiou, A.N. Lasenby, General Relativity, Cambridge University Press (2007).

Weekly Breakdown		
Week	***Section***	***Topics***
1	5.1-5.7	Special relativity revisited: Minkowski spacetime in Cartesian coordinates, Lorentz transformations, Cartesian basis vectors, Four-vectors and the lightcone, Four-vectors and Lorentz transformations, Four-velocity, Four-momentum of a massive particle.
2	5.8-5.14	Four-momentum of a photon, The Doppler effect and relativistic aberration, Relativistic mechanics, Free particles, Relativistic collisions and Compton scattering, Accelerating observers, Minkowski space time in arbitrary coordinates.
3	6.1-6.4	Electromagnetism: The electromagnetic force on a moving charge, The 4-current density, The electromagnetic field equations, Electromagnetism in the Lorenz gauge.
4	6.5-6.7	Electric and magnetic fields in inertial frames, Electromagnetism in arbitrary coordinates, Equation of motion for a charged particle, Electromagnetism in a curved spacetime.
5	8.1-8.7	The gravitational field equations: The energy–momentum tensor, The energy–momentum tensor of a perfect fluid, Conservation of energy and momentum for a perfect fluid, The Einstein equations, The Einstein equations in empty space, The weak-field limit of the Einstein equations, The cosmological-constant term.
6	9.1-9.7	The Schwarzschild geometry: General static isotropic metric, Schwarzschild solution, Birkhoff's theorem, Gravitational redshift, geodesics in Schwarzschild geometry, radial trajectories of massive particles.
7	9.8-9.13	Circular motion of massive particles, stability of massive particle orbits, trajectories of photons, circular motion of photons, stability of photon orbits.
8	10.1, 10.2, 10.4	Experimental tests of general relativity: Precession of planetary orbits, The bending of light, Accretion discs around compact objects.
9	**Mid Semester Exam**	
10	11.1 - 11.6	Schwarzschild black holes: singularities in Schwarzschild metric, radial photon worldlines, radial particle worldliness in Schwarzschild coordinates, Eddington Finkelstein coordinates, black hole formation.
11	11.7 - 11.11	Spherically symmetric collapse of dust, tidal forces near a black hole, Kruskal coordinates, wormholes and Einstein Rosen bridge, The Hawking effect of black hole evaporation.
12	12.1-12.6	Further spherically symmetric geometries: Spherically symmetric geometries: metric for stellar interior, relativistic equations of stellar structure, Schwarzschild interior solution, metric outside a spherically symmetric charged mass, Riessner-Nordstrom geometry and solution
13	12.7-12.8 13.5, 13.6	Radial photon trajectories in RN geometry, radial massive particle trajectories, Kerr metric: The Kerr metric, Limits of the Kerr metric, Ker Neumann Metric (handouts).
14	14.1-14.7	The Friedmann–Robertson–Walker geometry: The cosmological principle, synchronous commoving coordinates, homogeneity and isotropy of the universe, maximally symmetric 3-space, Friedmann-Robertson-Walker metric, geometrical properties of FRW metric.
15	14.9, 14.10	The cosmological redshift, The Hubble and deceleration parameters.
16	15.1-15.6	Components of the cosmological fluid, Cosmological parameters, The cosmological field equations, General dynamical behaviour of the universe, Evolution of the scale factor, Analytical cosmological models.
17	-	Review

18	**End Semester Exam**

MATH-956 Finite Volume Method

Credit Hours: 3-0
Prerequisite: None

Course Objectives: This course aims on a powerful class of numerical methods for approximating solution of hyperbolic partial differential equations, including both linear problems and nonlinear conservation laws.

Core Contents: Conservation laws, Finite volume methods, Multidimensional problems. Linear waves and discontinuous media. The advection equation. Diffusion and the advection–diffusion equation, Nonlinear equations in fluid dynamics. Linear acoustics, Sound waves. Hyperbolicity of linear systems, Variable-coefficient hyperbolic systems. Solution to the Cauchy problem. Superposition of waves and characteristic variables, Left eigenvectors, Simple waves, Acoustics, Domain of dependence and range of influence. Discontinuous solutions, The Riemann problem for a linear system. Coupled acoustics and advection, Initial–boundary-value problems. General formulation for conservation laws, A numerical flux for the diffusion equation, Necessary components for convergence, The CFL condition. An unstable flux, The Lax– Friedrichs method, The Richtmyer two-step Lax–Wendroff method, Upwind methods, The upwind method for advection. Godunov's method for linear systems, The numerical flux function for Godunov's method. Flux-difference vs. flux-vector splitting, Roe's method. The Lax–Wendroff method, The beam–warming method, Preview of limiters. Choice of slopes, Oscillations, Total variation. Slope-limiter methods, Flux formulation with piecewise linear reconstruction, Flux limiters, TVD limiters

Course Outcomes: Students are expected to understand the various variants of the of finite volume method and its applications to problems like:

- Linear waves and discontinuous media.
- Diffusion and the advection–diffusion equation.
- Coupled acoustics and advection.

Text Book: Randall J. Leveque, Finite Volume Methods for Hyperbolic, Problems, Cambridge University Press, (2004)

Reference Books: F. Moukalled, L. Mangani, M. Darwish, "The Finite Volume Method in Computational Fluid Dynamics'', Springer, 2016

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1-1.3	Conservation laws, Finite volume methods, Multidimensional problems.
2	1.4.2.1	Linear waves and discontinuous media. The advection equation.
3	2.2,2.6	Diffusion and the advection–diffusion equation, Nonlinear equations in fluid dynamics.
4	2.7,2.8	Linear acoustics, Sound waves.
5	2.9,2.10,3.1	Hyperbolicity of linear systems, Variable-coefficient hyperbolic systems. Solution to the Cauchy problem.
6	3.2-3.6	Superposition of waves and characteristic variables, Left eigenvectors, Simple waves, Acoustics, Domain of dependence and range of influence.
7	3.7,3.8	Discontinuous solutions, The Riemann problem for a linear system
8	3.10,3.11	Coupled acoustics and advection, Initial–boundary-value problems.
9	**Mid Semester Exam**	
10	4.1-4.4	General formulation for conservation laws, A numerical flux for the diffusion equation, Necessary components for convergence, The CFL condition.
11	4.5-4.9	An unstable flux, The Lax–Friedrichs method, The Richtmyer two-step Lax–Wendroff method, Upwind methods, The upwind method for advection.
12	4.10,4.11	Godunov's method for linear systems, The numerical flux function for Godunov's method.
13	4.13, 4.14	Flux-difference vs. flux-vector splitting, Roe's method
14	6.1-6.3	The Lax–Wendroff method, The beam–warming method, Preview of limiters.
15	6.5-6.7	Choice of slopes, Oscillations, Total variation.
16	6.9-6.12	Slope-limiter methods, Flux formulation with piecewise linear reconstruction, Flux limiters, TVD limiters
17	-	Review
18	**End Semester Exam**	

MATH-957 Algebraic Topology

Credit Hours: 3-0
Prerequisite: None

Course Objectives: The objective of this course is to introduce the basic concepts about homotopy and homotopy type, fundamental group and covering spaces to use in his/her research and in other areas like differential geometry, algebraic geometry, physics etc.

Core Contents: Connected spaces, Path connected spaces, Compact spaces, Homotopy equivalence, Path homotopy, Fundamental group, Induced homomorphism, Van Kampen's Theorem, Covering spaces, Singular homology, Homotopy invariance, Homology long exact sequence.

Detailed Course Contents: Topological spaces, Closure and interior points, Bases, Continuity, Homeomorphism, Compactness, Path connectedness, Connectedness, Relationship between connectedness and path connectedness, History of algebraic topology, Homotopy, Homotopy classes, Path homotopy, Fundamental group, Fundamental group of a circle, Induced homomorphism, Van Kampen's theorem, Covering space, Universal cover, Classification of Covering spaces, Deck transformation, Covering space action, Idea of Homology, Simplicial homology, Singular homology, Chain homotopy, Homotopy invariance of Homology, Exact sequence, Degree and Cellular homology, Application of homology in group.

Learning Outcomes: On successful completion of this course students will be able to:

- Understand the definitions of homotopy, homotopy equivalence, fundamental group.
- Understand methods to construct and classify covering spaces for known spaces, and for other spaces whenever it is possible.
- Understand the relation between singular homology and fundamental group.
- Understand the homology of a group.

Textbooks:
Andrew H. Wallace, (AW) "An Introduction to Algebraic Topology", Dover Publisher, (2007)
Allen Hatcher, (AH) "Algebraic Topology", Cambridge University Press, (2002)

Reference Books:

1. Joseph J. Rotman, "An Introduction to Algebraic Topology", Springer, (1988)
2. J. Peter May, "A Concise Course in Algebraic Topology", Chicago University Press, (1999)
3. R. Brown, "Topology and Groupoids", BookSurge Publishing, (2006)

Weekly Breakdown		
Week	***Section***	***Topics***
1	(AW) 2.1-2.8	Definition of Topology, Open sets, Subspace, Limit and Closure points, Bases
2	3.1-3.2	Continuous Mapping, Homeomorphism, Compactness
3	3.3	Pathwise Connectedness and Related Results
4	3.4	Connectedness, Examples, Relationship between Connectedness and Pathwise connectedness
5	4.1	History of Algebraic Topology, Homotopy and Results, Homotopy
6	4.2	Homotopy classes, Path Homotopy and Results
7	4.3-4.4	Fundamental Groups, Fundamental group of a Circle
8	(AH) 1.1.3	Induced Homomorphism and Results
9	**Mid Semester Exam**	
10	1.2.1, 1.2.2	Free Product of Groups, Van Kampen's theorem and Application
11	1.3.1, 1.3.2	Covering Spaces and Lifting Criterion, Universal Cover
12	1.3.3	Classification of Covering space, Deck Transformation and Group actions
13	2.1.1, 2.1.2	Homology, Types of Homology, Simplicial Homology
14	2.1.3	Singular Homology, Chain Homotopy
15	2.1.4-2.1.5	Homotopy invariance of Homology, Exact Sequence
16	2.2.1-2.2.2,	Degree and Cellular homology, Homology of a group
17	-	Review
18	**End Semester Exam**	

MATH-XXX Finite Difference Methods for Differential Equations

Credit Hours: 3-0
Prerequisite: None

Course Objectives: The objective of this course is to find numerical solution of ordinary and partial differential equations by finite difference method. The basics and advanced topics relevant to finite difference method will be covered. These topics will be very useful for the students who opts for the research topic in the area of differential equations. Not only students will be given theoretical aspects of numerical schemes but also programming experience in MATLAB will be helpful.

Core contents: Finite difference approximations, boundary value problems, elliptic equations, iterative method for sparse system, advection equations and hyperbolic systems

Course Contents: Truncation errors, finite difference approximations, the heat equation, the steady-state problem, local truncation error, global error, stability, consistency, steady-state heat conduction, Jacobi and Gauss-Seidal, rate of convergence, The Arnoldi process and GMRES algorithm, Advection equation, Leapfrog method, Lax-Friedrichs, The Lax-Wendroff method, Upwind methods, Von Neumann analysis, The Courant-Friedrichs-Lewy condition

Course Outcomes: After studying this subject the students will be able to:

- Compute numerical solution of ODEs and PDEs by finite difference method
- Solve sparse linear system by iterative schemes
- Program numerical solutions in MATLAB

Textbook: Finite Difference Methods for Ordinary and Partial Differential Equations by Randall J. LeVeque, Publisher: Siam, 2007.

Reference Books:

1. Applied Numerical Analysis by Curtis F. Gerald and Patrick O. Wheatley, 7th Edition, Publisher: Pearson, 2003.
2. Numerical Methods for Engineers by Steven C Chapra and Raymond P Canale, 6th Edition, Publisher: McGraw-Hill, 2009.
3. Finite Difference Computing with PDEs: A Modern Software Approach by Hans Petter Langtangen and Svein Linge, Ist Edition, Publisher: Springer, 2017.

Weekly Breakdown		
Week	*Sections*	*Topic*
1	1.1, 1.2, 1.3	Truncation errors, Deriving finite difference approximations, Second order derivatives
2	2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 2.10	The heat equation, Boundary conditions, The steady-state problem, A simple finite difference method, Local truncation error, Global error, Stability, Consistency, Convergence, Stability in the 2- norm
3	2.15, 2.16, 2.16.1, 2.16.2, 2.16.3,	A general linear second order equation, Nonlinear equations, Discretization of the nonlinear boundary value problem, Nonuniqueness, Accuracy on nonlinear equations
4	3.1, 3.2, 3.3	Steady-state heat conduction, The 5-point stencil for the Laplacian, Ordering the unknowns and equations
5	3.4, 3.5, 3.6, 3.7, 3.7.1	Accuracy and stability, The 9-point Laplacian, Other elliptic equations, Solving the linear system, Spare storage in MATLAB
6	4.1, 4.2, 4.2.1, 4.2.2	Jacobi and Gauss-Seidal, Analysis of matrix slitting methods, Rate of convergence, Successive overrelaxation
7	4.4 4.4.1 4.4.2	The Arnoldi process and GMRES algorithm, Krylov methods based on three term recurrences, Other applications of Arnoldi
8	4.5	Newton-Krylov methods for nonlinear problems
9	**Mid Semester Exam**	
10	4.6	Multigrid methods
11	4.6.1, 4.6.2	Slow convergence of Jacobi, The multigrid approach
12	10.1 10.2 10.2.1	Advection, Method of lines discretization, Forward Euler time discretization,
13	10.2.2 10.2.3	Leapfrog, Lax-Friedrichs
14	10.3 10.3.1 10.4 10.4.1 10.4.2	The Lax-Wendroff method, Stability Analysis, Upwind methods, Stability Analysis, The Beam-Warming method
15	10.5 10.6 10.7	Von Neumann analysis, Characteristic tracing and interpolation, The Courant-Friedrichs-Lewy condition
16	10.8 10.9 10.10	Some numerical results, Modified equations, Hyperbolic systems
17	-	Review
18	**End Semester Exam**	

Annex C

Revised List of MS Mathematics Courses with Prerequisites

Core Courses

S. No	Course Code	Course Title	Credits	Prerequisite
1	MATH-801	Algebra	3-0	None
2	MATH-803	Geometry	3-0	None
3	MATH-XXX	Computational Mathematics	3-0	None
4	MATH-817	Advanced Functional Analysis	3-0	None

Elective Course

S. No	Course Code	Course Title	Credits	Prerequisite
1.	MATH-807	Commutative Algebra	3-0	None
2.	MATH-818	Theory of Ordinary Differential Equations	3-0	None
3.	MATH-819	Analysis of Fractional Differential Equations	3-0	None
4.	MATH-820	Calculus of Variations and Optimal Control	3-0	None
5.	MATH-821	Analytical Approximate Solutions of ODEs	3-0	None
6.	MATH-822	Mathematical Modelling-I	3-0	None
7.	MATH-XXX	Advanced Topology	3-0	None
8.	MATH-903	Partial Differential Equations-I	3-0	None
9.	MATH-905	Symmetry Methods for Differential Equations-I	3-0	None
10.	MATH-908	Fixed Point Theory	3-0	None
11.	MATH-909	Continuum Mechanics-I	3-0	None
12.	MATH-911	Special Functions	3-0	None
13.	MATH-941	Graph Theory	3-0	None
14.	MATH-943	Convex Analysis	3-0	None
15.	MATH-944	Semigroup Theory of Operators	3-0	None
16.	MATH-945	Lie Group Representations	3-0	None
17.	MATH-946	Category Theory	3-0	None
18.	MATH-949	Combinatorics	3-0	None
19.	MATH-955	General Relativity and Cosmology	3-0	None
20.	MATH-956	Finite Volume Method	3-0	None
21.	MATH-957	Algebraic Topology	3-0	None
22.	MATH-XXX	Finite Difference Methods for Differential Equations	3-0	None
23.	PHY-801	Classical Mechanics	3-0	None
24.	PHY-803	Quantum Mechanics	3-0	None
25.	PHY-805	Electromagnetism	3-0	None
26.	PHY-806	Thermal Physics	3-0	None
27.	PHY-902	Quantum Field Theory-I	3-0	None
28.	PHY-907	General Relativity	3-0	None
29.	PHY-908	Cosmology-I	3-0	None

30.	PHY-912	Relativistic Astrophysics	3-0	None
31.	PHY-914	Particle Physics-I	3-0	None
32.	PHY-920	Classical Field Theory	3-0	None
33.	STAT-806	Statistical Learning	3-0	None
34.	ME-881	Advanced Fluid Mechanics	3-0	None

Annex D

Revised List of Ph.D. Mathematics Courses with Prerequisites

S. No	Course Code	Course Title	Credits	Prerequisite
1.	MATH-801	Algebra	3-0	None
2.	MATH-803	Geometry	3-0	None
3.	MATH-XXX	Computational Mathematics	3-0	None
4.	MATH-807	Commutative Algebra	3-0	None
5.	MATH-817	Advanced Functional Analysis	3-0	None
6.	MATH-818	Theory of Ordinary Differential Equations	3-0	None
7.	MATH-819	Analysis of Fractional Differential Equations	3-0	None
8.	MATH-820	Calculus of Variations and Optimal Control	3-0	None
9.	MATH-821	Analytical Approximate Solutions of ODEs	3-0	None
10.	MATH-822	Mathematical Modelling-I	3-0	None
11.	MATH-XXX	Advanced Topology	3-0	None
12.	MATH-903	Partial Differential Equations-I	3-0	None
13.	MATH-904	Partial Differential Equations-II	3-0	MATH-903 Partial Differential Equations-I
14.	MATH-905	Symmetry Methods for Differential Equations-I	3-0	None
15.	MATH-906	Symmetry Methods for Differential Equations-II	3-0	MATH-905 Symmetry Methods for Differential Equations-I
16.	MATH-908	Fixed Point Theory	3-0	None
17.	MATH-909	Continuum Mechanics-I	3-0	None
18.	MATH-910	Continuum Mechanics-II	3-0	MATH-909 Continuum Mechanics-I
19.	MATH-911	Special Functions	3-0	None
20.	MATH-941	Graph Theory	3-0	None
21.	MATH-943	Convex Analysis	3-0	MATH-802 Analysis
22.	MATH-944	Semigroup Theory of Operators	3-0	None
23.	MATH-945	Lie Group Representations	3-0	None
24.	MATH-946	Category Theory	3-0	None
25.	MATH-949	Combinatorics	3-0	None
26.	MATH-951	Mathematical Modelling-II	3-0	MATH-822Mathematical Modelling-I
27.	MATH-955	General Relativity and Cosmology	3-0	None
28.	MATH-956	Finite Volume Method	3-0	None
29.	MATH-957	Algebraic Topology	3-0	None
30.	MATH-XXX	Finite Difference Methods for Differential Equations	3-0	None
31.	PHY-801	Classical Mechanics	3-0	None
32.	PHY-803	Quantum Mechanics	3-0	None

33.	PHY-805	Electromagnetism	3-0	None
34.	PHY-806	Thermal Physics	3-0	None
35.	PHY-902	Quantum Field Theory-I	3-0	None
36.	PHY-907	General Relativity	3-0	None
37.	PHY-908	Cosmology-I	3-0	None
38.	PHY-912	Relativistic Astrophysics	3-0	None
39.	PHY-914	Particle Physics-I	3-0	None
40.	PHY-920	Classical Field Theory	3-0	None
41.	ME-881	Advanced Fluid Mechanics	3-0	None
42.	MATH-960	Reading and Research-I	3-0	None
43.	MATH-961	Reading and Research-II	3-0	None
44.	MATH-982	Seminar Delivered-G*	0	-
45.	MATH-984	Seminar Delivered-R*	0	-
46.	SEM/WKSP-997	Seminar/Workshop**	1-0	-
47.	MATH-999	PhD Thesis	30	-

*Additional Course
**Additional Course (Seminar/Workshop attendance)